Econ 345: Money and Banking | University of British Columbia
Oleksiy Kryvtsov | Winter 2009-2010, Term 2

Homework 1
Answer Key

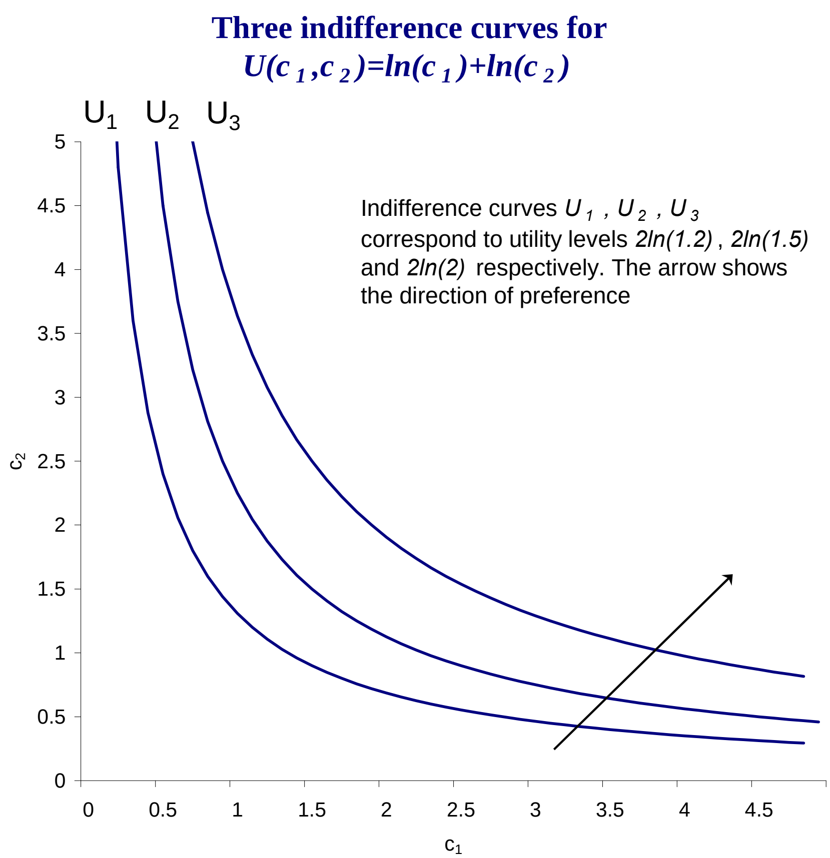
Show all your work. Please type or use a pen. Assignments written in pencil will not be regraded. Graphs should be clearly labeled. The total for the homework is 5 points. Start early. Homework is due in class on Wednesday, January 20 (or if you cannot make it to class, drop it off in the main office, Buchanan Tower 997, by the end of the class). No late homeworks are excepted.

Problem 1 (1 point)

In class we defined an indifference curve as a line connecting consumption bundles that provide same utility to an individual. In this problem you are asked to experiment with a particular utility function.

Assume the following utility function: $U(c_1, c_2) = \ln(c_1) + \ln(c_2)$. An indifference curve corresponds to a curve on (c_1,c_2) diagram for a given level of utility $U(c_1,c_2)$. For example, the indifference curve from Lecture 2 corresponds to utility level of $2\ln(1.5)$, so that the line is given by equation $\ln(c_1) + \ln(c_2) = 2\ln(1.5)$.

(a) For the utility above, pick three different utility levels (three numbers) and draw three corresponding indifference curves. Draw an arrow showing the direction of preference.



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(b) Rank the following consumption bundles in terms of utility derived from them. (HINT: you do not need a computer to be able to do this exercise, just use the three properties of indifference curves that we learned in class: two points on the same indifference curve are equally preferred, more consumption is better, and more balanced consumption is better):
$(c_1, c_2) =$ (2,1); (6,4); (1,0); (5,5); (1,1.5); (1.5, 8.5); (1.5, 1.5)

ANSWER: In the order of increasing utility: (1,0); (1,1.5); (2,1); (1.5, 1.5); (1.5, 8.5); (6,4); (5,5).

(1,1.5) is strictly preferred to (1,0) because it gives more of c_2 (keeping c_1 constant)

(2,1) is strictly preferred to (1,1.5) because it is equally preferred to (1,2) which is strictly preferred to (1,1.5)

(1.5, 1.5) is strictly preferred to (2,1) because it is more balanced than (2,1) (both sum up to consumption of 3 units of consumption good)

(1.5, 8.5) is strictly preferred to (1.5, 1.5) because it gives more of c_2 (keeping c_1 constant)

(6,4) is strictly preferred to (1.5, 8.5) because it is more balanced than (1.5, 8.5) (both sum up to consumption of 10 units of consumption good)

(5,5) is strictly preferred to (6,4) because it is more balanced than (6, 4) (both sum up to consumption of 10 units of consumption good)

ALTERNATIVE ANSWER: Calculate utility level for each bundle and order them in the order of utility level:

$U(1,0) = -\infty$
$< U(1,1.5) = 0.406$
$< U(2,1) = 0.693$
$< U(1.5,1.5) = 0.811$
$< U(1.5,8.5) = 2.546$
$< U(6,4) = 3.175$
$< U(5,5) = 3.219$

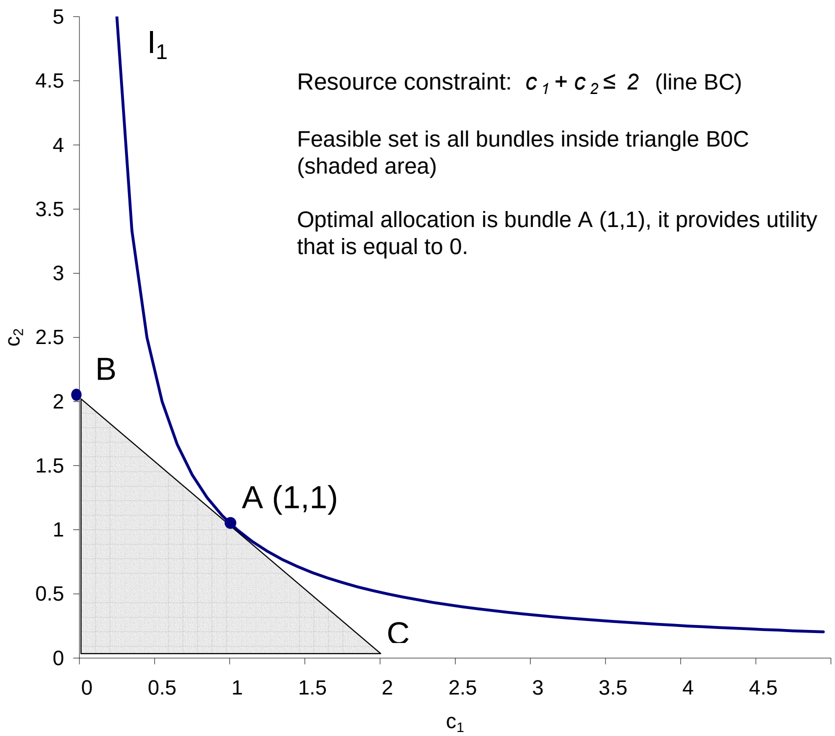
Problem 2 (2 points)

Consider OLG economy with constant population N=100 and endowments of 2 units of consumption good for young and 0 units for old. Assume that initial old are endowed with $M=3$ dollars of fiat money. For all questions, consider only stationary consumption allocation: $c_{1,t}=c_1$ and $c_{2,t}=c_2$ in all periods t.

(a) Draw a resource constraint. What is the set of feasible allocations?
(b) Assume preferences are given by the utility function from Problem 1. Find the optimal allocation on the diagram. (You should be able to find specific bundle, but it is not required).

ANSWER to (a),(b): See graph. The aggregate consumption must be less or equal than the aggregate endowment in any feasible allocation: $N_t c_{1,t} + N_{t-1} c_{2,t} \le N_t y$. *Since the population is constant we can get rid of Ns and time subscripts for c's and obtain the following per capita feasibility constraint:* $c_1 + c_2 \le y = 2$. *The feasible set line will simply be a line with negative 1 slope that has intercepts equal to y=2 on both vertical and horizontal axis. The feasible set itself will be all nonnegative consumption bundles below the feasibility line. Draw the indifference curve corresponding to utility level of zero. The point where the indifference curve touches the feasibility line determines the stationary combination of* c_1*, and* c_2 *that maximizes the utility of future generations. That point is (1,1), it is the optimal allocation.*

Resource constraint and optimal allocation for
U(c₁,c₂)=ln(c₁)+ln(c₂)* and *y=2



(c) Describe trades that are being made. Write down budget constraints for young and old for a given price. Derive the lifetime budget constraint.

ANSWER: In each period current young exchange c_2 *units of the good for 3 dollars from the current old. In period t young face budget constraint:* $c_1 + \frac{m_t}{p_t} \le 2$ *and old face the budget constraint:* $c_2 \le \frac{m_{t-1}}{p_t}$, *where* m_t *denotes money demand by the old (in equilibrium*

m_t must be equal to money supply of 3 dollars). Combining these two budget constraint gives lifetime budget constraints for generation born in period t : $c_1 + \frac{p_{t+1}}{p_t} c_2 \leq 2$

(d) Write down money market clearing condition (total money supply equals total money demand) and derive the rate of growth of prices (which is the inverse of the real return on money).

ANSWER: Total money demand in period t is how much money the current young demand in period t to finance their consumption purchases in period t+1 when they become old: $100\, c_2\, p_{t+1}$ *. Total money supply in period t is how much money there is in the economy in that period (all of it is held by old old born in t-1):* $N \cdot M = 100 \cdot 3 = 300$ *. Market clearing condition is then* $100\, c_2\, p_t = 300$ *. The growth rate of prices is then* $\frac{p_{t+1}}{p_t} = 1$ *. So prices are constant if total money supply is constant.*

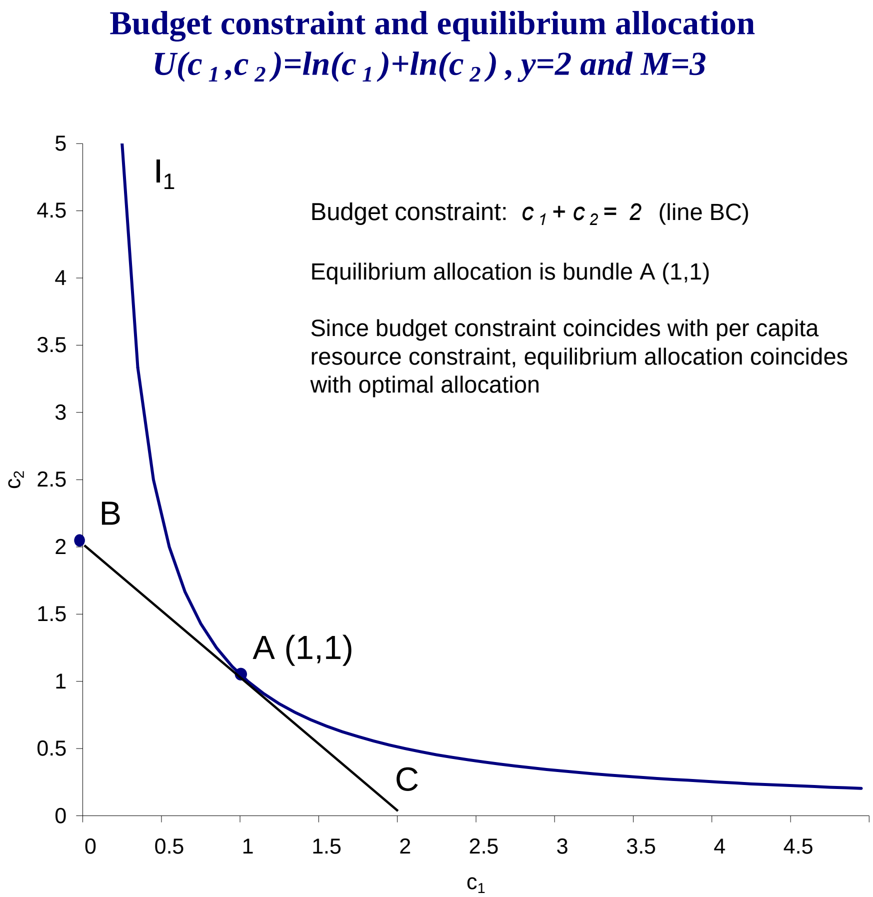
(e) Plug prices into budget constraint and draw it on the diagram.

ANSWER: Plug the growth rate of prices $\frac{p_{t+1}}{p_t} = 1$ *into lifetime budget constraint found in (c)* $c_1 + \frac{p_{t+1}}{p_t} c_2 \leq 2$ *to obtain* $c_1 + c_2 \leq 2$

(f) For preferences from Problem 1 what is the equilibrium stationary allocation? (Point on the diagram will do).

(g) Is this equilibrium allocation optimal? Explain.

ANSWERS for (f) and (g) :



(h) Define money velocity as total endowment in dollars divided by total money supply. Quantity theory of money predicts that velocity is constant – does that prediction hold in this economy?

ANSWER: Total endowment in dollars in period t is $p_t N \cdot y = 200\, p_t$. *Total money supply is* $N \cdot M = 300$. *So money velocity is* $V_t = \frac{2}{3} p_t$. *From (d) we know that prices are constant, so the velocity is constant as well. This prediction is consistent with Quantity theory of money.*

Problem 3 (2 points)

How would your answers in Problem 2 (a)-(h) change if you make the following alternative assumptions? If there is no change, you do not have to explain.

(a) Initial endowment is $M=6$ dollars.

ANSWER: Increasing the level of money endowment only increases the price level. All answers to (a)-(h) remain the same. Below I provide more details.

(a), (b) ***No change*** *since the resource constraint and preferences are the same as in Problem 2*

(c) Describe trades that are being made. Write down budget constraints for young and old for a given price. Derive the lifetime budget constraint.

ANSWER: In each period current young exchange c_2 units of the good for 6 dollars from the current old. In period t young face budget constraint: $c_1 + \frac{m_t}{p_t} \leq 2$ *and old face the budget constraint:* $c_2 \leq \frac{m_{t-1}}{p_t}$*, where m_t denotes money demand by the old (in equilibrium m_t must be equal to money supply of 6 dollars). Combining these two budget constraints gives lifetime budget constraint for generation born in period t :*

$c_1 + \frac{p_{t+1}}{p_t} c_2 \leq 2$. ***This is the same budget constraint as in Problem 2.***

(d) Write down money market clearing condition (total money supply equals total money demand) and derive the rate of growth of prices (which is the inverse of the real return on money).

ANSWER: Total money demand in period t is how much money the current young demand in period t to finance their consumption purchases in period t+1 when they become old: $100\ c_2\ p_{t+1}$. Total money supply in period t is how much money there is in the economy in that period (all of it is held by old born in t-1): $N \cdot M = 100 \cdot 6 = 600$. Market clearing condition is then $100\ c_2\ p_t = 600$. The growth rate of prices is then $\frac{p_{t+1}}{p_t} = 1$. ***So prices are constant if total money supply is constant.***

(e) Plug prices into budget constraint and draw it on the diagram.
(f) For preferences from Problem 1 what is the equilibrium stationary allocation? (Point on the diagram will do).
(g) Is this equilibrium allocation optimal? Explain.

ANSWERS for (e)-(g) : ***No change*** *since prices are still constant and budget constraint is the same as in Problem 2.*

(h) Define money velocity as total endowment in dollars divided by total money supply. Quantity theory of money predicts that velocity is constant – does that prediction hold in this economy?

ANSWER: Total endowment in dollars in period t is $p_t\ N \cdot y = 200\ p_t$. Total money supply is $N \cdot M = 600$. So money velocity is $V_t = \frac{1}{3} p_t$*. From (d) we know that prices* *are constant, so the* ***velocity is constant*** *as well. This prediction is consistent with Quantity theory of money.*

(b) Population is growing at a rate of 10% per period. Total money supply is constant at 300 dollars.

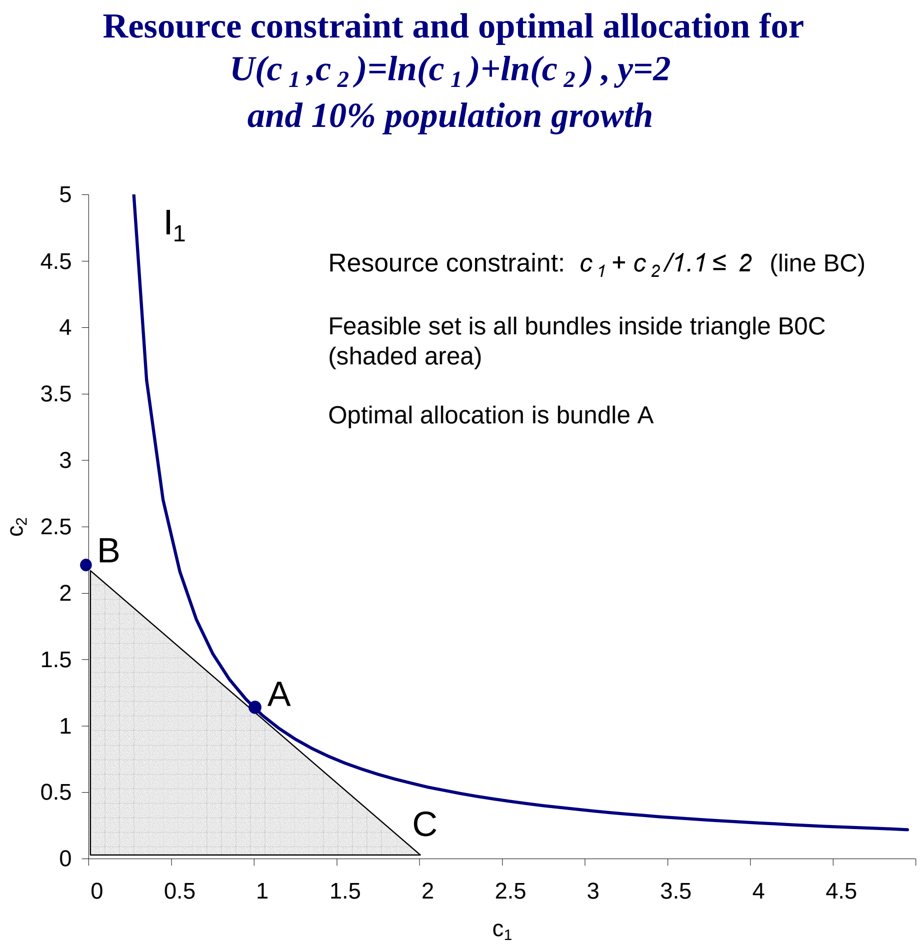
ANSWERS:

(a) Draw a resource constraint. What is the set of feasible allocations?
(b) Assume preferences are given by the utility function from Problem 1. Find the optimal allocation on the diagram. (You should be able to find specific bundle, but it is not required).

ANSWER to (a),(b): See graph. The aggregate consumption must be less or equal than the aggregate endowment in any feasible allocation: $N_t c_{1,t} + N_{t-1} c_{2,t} \leq N_t y$.

Since the population is growing at a constant rate, $\frac{N_t}{N_{t-1}} = 1.1$*, we can divide the constraint by* N_{t-1} *and obtain the following per capita feasibility constraint:*

$c_1 + \frac{c_2}{1.1} \leq 2$. *The feasible set line will simply be a line that connects points (2,0) and (0,2.2). The feasible set itself will be all nonnegative consumption bundles below the feasibility line. The point where the indifference curve touches the feasibility line determines the stationary combination of* c_1*, and* c_2 *that maximizes the utility of future generations.*



(c) Describe trades that are being made. Write down budget constraints for young and old for a given price. Derive the lifetime budget constraint.

ANSWER: **No change.** *In each period current young exchange c_2 units of the good for 3 dollars from the current old. In period t young face budget constraint:* $c_1 + \frac{m_t}{p_t} \leq 2$ *and old face the budget constraint:* $c_2 \leq \frac{m_{t-1}}{p_t}$*, where m_t denotes money demand by the old (in equilibrium m_t must be equal to money supply of 3 dollars). Combining these two budget constraints gives lifetime budget constraint for generation born in period t :* $c_1 + \frac{p_{t+1}}{p_t} c_2 \leq 2$

(d) Write down money market clearing condition (total money supply equals total money demand) and derive the rate of growth of prices (which is the inverse of the real return on money).

ANSWER: Total money demand in period t is how much money the current young demand in period t to finance their consumption purchases in period t+1 when they become old: $N_t\, c_2\, p_{t+1}$ *. Total money supply in period t is how much money there is in the economy in that period (all of it is held by old): 300 dollars . Market clearing condition is then* $N_{t-1}\, c_2\, p_t = 300$ *. The growth rate of prices is then* $\frac{p_{t+1}}{p_t} = 1/1.1$ *.*

So prices are falling at the rate of population growth.

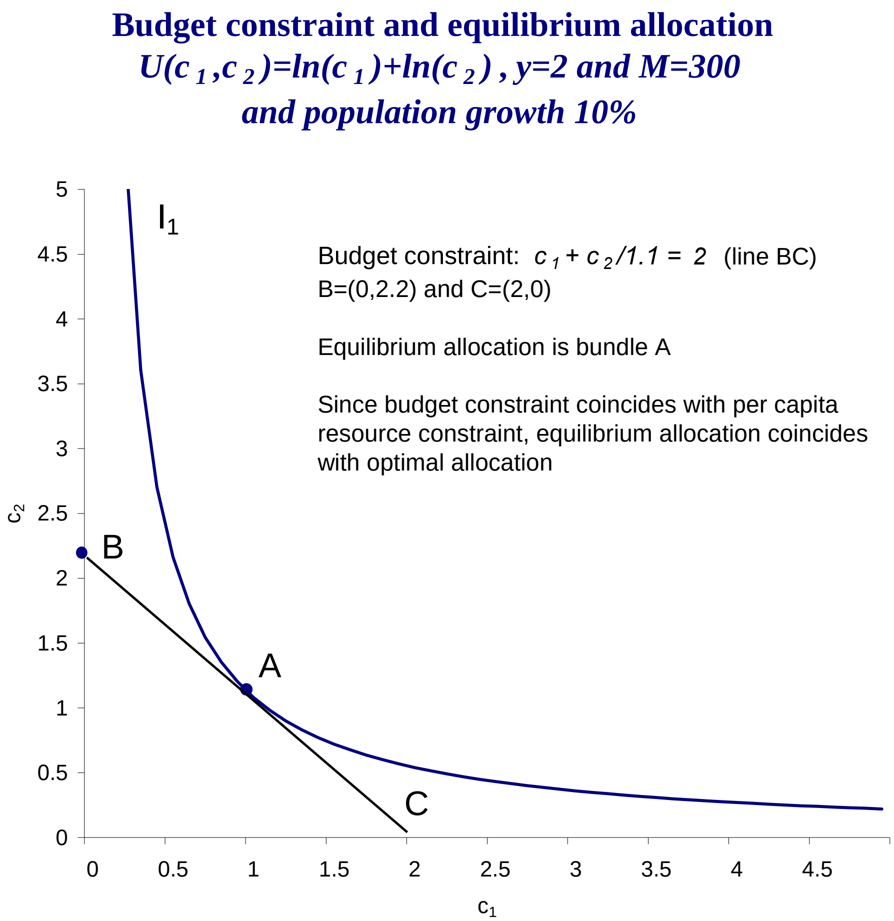
(e) Plug prices into budget constraint and draw it on the diagram.

ANSWER: Plug the growth rate of prices $\frac{p_{t+1}}{p_t} = 1/1.1$ *into lifetime budget constraint found in (c)* $c_1 + \frac{p_{t+1}}{p_t} c_2 \le 2$ *to obtain* $c_1 + c_2 / 1.1 \le 2$

(f) For preferences from Problem 1 what is the equilibrium stationary allocation? (Point on the diagram will do).

(g) Is this equilibrium allocation optimal? Explain.

ANSWERS for (f) and (g) :



(h) Define money velocity as total endowment in dollars divided by total money supply. Quantity theory of money predicts that velocity is constant – does that prediction hold in this economy?

ANSWER: Total endowment in dollars in period t is $p_t N_{t-1} \cdot y = 2N_{t-1} p_t$. *Total money supply is 300. So money velocity is* $V_t = \frac{2N_{t-1}}{300} p_t$. *From (d) we know that prices are falling at the rate of population growth, so* $N_{t-1}p_t$ *is constant and the velocity is constant as well.* ***This prediction is consistent with Quantity theory of money.***

(c) Money supply is growing at a rate of 20% per period (for example, in every period central bank gives current old extra amount of cash so that total cash in the economy is growing at 20%).

(a), (b) ***No change*** *since the resource constraint and preferences are the same as in Problem 2*

(c) Describe trades that are being made. Write down budget constraints for young and old for a given price. Derive the lifetime budget constraint.

ANSWER: In each period current young exchange c_2 units of the good for m_t dollars from the current old. In period t young face budget constraint: $c_1 + \frac{m_t}{p_t} \le 2$ *and old face the budget constraint:* $c_2 \le \frac{m_{t-1}}{p_t}$*, where m_t denotes money demand by the old (in equilibrium m_t must be equal to money supply). Combining these two budget constraints gives lifetime budget constraint for generation born in period t :*
$c_1 + \frac{p_{t+1}}{p_t} c_2 \le 2$. ***This is the same budget constraint as in Problem 2.***

(d) Write down money market clearing condition (total money supply equals total money demand) and derive the rate of growth of prices (which is the inverse of the real return on money).

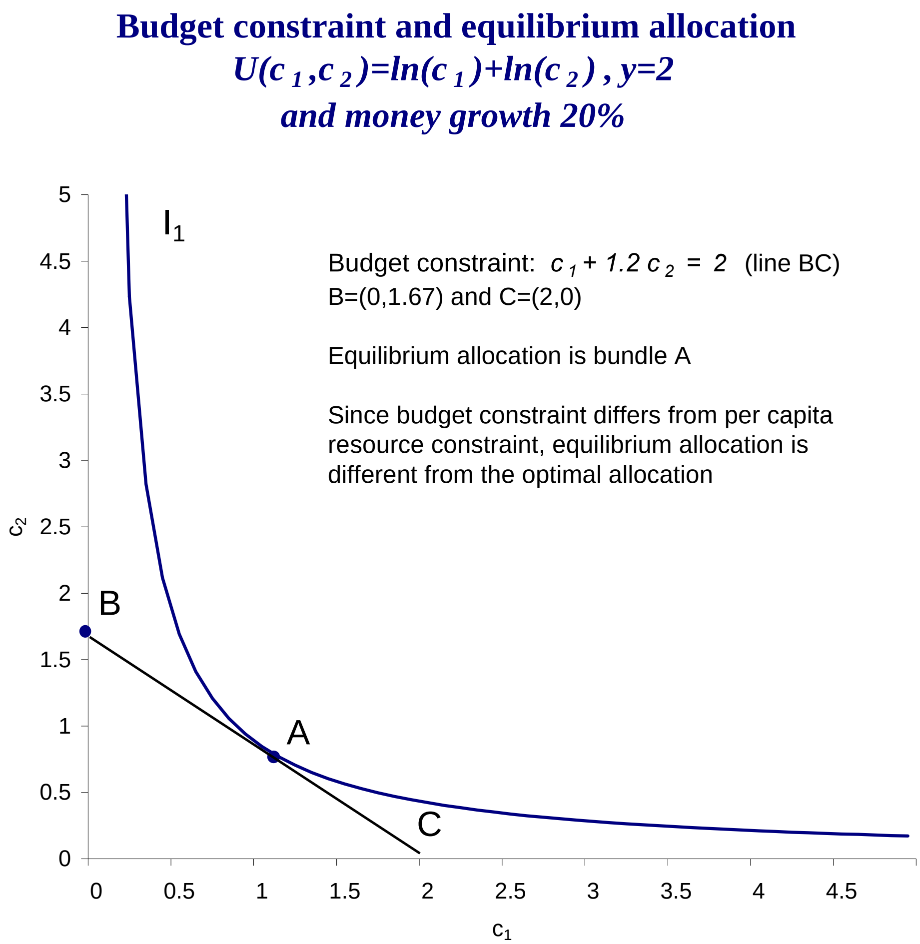
ANSWER: Total money demand in period t is how much money the current young demand in period t to finance their consumption purchases in period t+1 when they become old: 100 c_2 p_{t+1} . Total money supply in period t is how much money there is in the economy in that period (all of it is held by old born in t-1): M_t . Market clearing condition is then 100 c_2 p_t = M_t . The growth rate of prices is then
$\frac{p_{t+1}}{p_t} = \frac{M_{t+1}}{M_t} = 1.2$. ***So prices grow at the rate of money growth.***

(e) Plug prices into budget constraint and draw it on the diagram.

ANSWER: Plug the growth rate of prices $\frac{p_{t+1}}{p_t} = 1.2$ *into lifetime budget constraint found in (c)* $c_1 + \frac{p_{t+1}}{p_t} c_2 \le 2$ *to obtain* $c_1 + 1.2 c_2 \le 2$. *This is the line connecting points (2,0) and (0,1.67). See graph.*

(f) For preferences from Problem 1 what is the equilibrium stationary allocation? (Point on the diagram will do).
(g) Is this equilibrium allocation optimal? Explain.

ANSWERS for (e)-(g) :



(h) Define money velocity as total endowment in dollars divided by total money supply. Quantity theory of money predicts that velocity is constant – does that prediction hold in this economy?

ANSWER: Total endowment in dollars in period t is $p_t N \cdot y = 200\, p_t$. *Total money supply is* M_t. *So money velocity is* $V_t = \frac{200}{M_t} p_t$. *From (d) we know that prices are growing at the rate of money growth, so* p_t/M_t *is constant and the* ***velocity is constant*** *as well. This prediction is consistent with Quantity theory of money.*

(d) It is known that in period 35 central bank gives the old an infinite amount of money.

ANSWER: In period 34 young will not be willing to sell part of their endowment to old for cash because they know that in period 35 when the become old and money has no value – they will not be able to buy consumption. So there is no trade in period 34. Similarly, in period 33 young know that next period young will not accept the cash so they will not trade too, leading to no trade in period 33. Continuing this logic for periods

32, 31, …, 1, we conclude that equilibrium in the economy when money is not valued in one of the periods is an autarky (no trade equilibrium).