Appendix 1: Simple Calvo Economy

Equilibrium system of equations

$$\begin{aligned} c_t &= s_t \\ y_t &= s_t + inv_t \\ inv_t &= z_t - s_t - (1-\delta)\left(z_{t-1} - s_{t-1}\right) \end{aligned}$$

$$W_t = W_{t-1}^{\lambda}\left(P_t c_t\right)^{1-\lambda}$$

$$\begin{aligned} M_t &= P_t c_t \\ \mu_t &= M_t / M_{t-1} \end{aligned}$$

$$v_{t+j}^{*}(i) = \frac{z_{t+j}(i)}{s_{t+j}}\left(\frac{P_t(i)}{P_{t+j}}\right)^{\theta}$$

$$MC_t = W_t$$

$$\Lambda_t = MC_t$$

$$1 - F(v_{t+j}^{*}(i)) = \frac{\Lambda_{t+j} - (1-\delta)E_{t+j}\frac{\beta U_{t+j+1}P_{t+j}}{U_{t+j}P_{t+j+1}}\Lambda_{t+j+1}}{P_t(i) - (1-\delta)E_{t+j}\frac{\beta U_{t+j+1}P_{t+j}}{U_{t+j}P_{t+j+1}}\Lambda_{t+j+1}}$$

$$\begin{aligned} &E_t \sum_{j=0}^{\infty} \xi_p^j \frac{\beta^j U_{t+j} P_t}{U_t P_{t+j}} \frac{Z_{t+j}(i)}{\omega_{t+j}(i)} \left\{(1-\theta)P_t(i) + \theta(1-\delta)E_{t+j}\frac{\beta U_{t+j+1}P_{t+j}}{U_{t+j}P_{t+j+1}}\Lambda_{t+j+1}\right. \\ &\left. +\theta\omega_{t+j}(i)\left[\Lambda_{t+j} - (1-\delta)E_{t+j}\frac{\beta U_{t+j+1}P_{t+j}}{U_{t+j}P_{t+j+1}}\Lambda_{t+j+1}\right]\right\} = 0 \end{aligned}$$

$$P_t^{1-\theta} = \int_0^1 P_t^{1-\theta}(i)\, \Phi(v_t^*(i))di$$

Normalization

$$w_t = W_t/P_t,\ m_t = M_t/P_t,\ \text{etc.}$$

Stationary equations

$$inv_t = z_t - s_t - (1-\delta)(z_{t-1} - s_{t-1})$$

$$c_t = s_t$$

$$s_t + inv_t = y_t$$

$$w_t = w_{t-1}^{\lambda} \pi_t^{-\lambda} c_t^{1-\lambda}$$

$$\begin{aligned} m_t &= c_t \\ \mu_t &= m_t/m_{t-1}\pi_t \end{aligned}$$

$$mc_t = w_t$$

$$\lambda_t = mc_t$$

$$v^*_{t+j}(i) = p_t^{\theta}(i)\,\frac{z_{t+j}(i)}{s_{t+j}}$$

$$1 - F(v^*_{t+j}(i)) = \frac{\lambda_{t+j} - (1-\delta)E_{t+j}\left(\frac{\beta c_{t+j}}{c_{t+j+1}}\lambda_{t+j+1}\right)}{p_t(i) - (1-\delta)E_{t+j}\left(\frac{\beta c_{t+j}}{c_{t+j+1}}\lambda_{t+j+1}\right)}$$

$$\begin{aligned}
&\frac{z_t(i)}{\omega_t(i)}(\theta-1)\left[p_t(i) - \frac{\theta}{\theta-1}\left\{\omega_t(i)\lambda_t + (1-\omega_t(i))(1-\delta)E_t\frac{\beta\xi_{t+1}}{\xi_t}\lambda_{t+1}\right\}\right] \\
&+E_t\sum_{j=1}^{\infty}\xi_p^j\frac{\beta^j c_t}{c_{t+j}}\frac{z_{t+j}(i)}{\omega_{t+j}(i)}(\theta-1)\times \\
&\times\left[p_t(i) - \frac{\theta}{\theta-1}\left\{\omega_{t+j}(i)\lambda_{t+j} + (1-\omega_{t+j}(i))(1-\delta)E_{t+j}\frac{\beta c_{t+j}}{c_{t+j+1}}\lambda_{t+j+1}\right\}\right] \\
=\;& 0
\end{aligned}$$

where $\omega_{t+j}(i) = \frac{v^*_{t+j}(i)}{\int \min(v, v^*_{t+j}(i))dF(v)}$.

$$\begin{aligned}
1 &= \int_0^1 p_t^{1-\theta}(i)\,\Phi(v_t^*(i))di \\
&= (1-\xi_p)p_t^{1-\theta}(i)\,\Phi(v_t^*(i)) + (1-\xi_p)\sum_{j=1}^{\infty}\xi_p^j p_{t-j}^{1-\theta}(i)\,\Phi(v_t^*(i))
\end{aligned}$$

Steady state

$$inv_* = \delta\,(z_* - s_*)$$

$$c_* = s_*$$

$$s_* + inv_* = y_*$$

$$w_* = \pi_*^{-\lambda} c_*$$

$$\begin{aligned} m_* &= c_* \\ \mu &= \pi_* \end{aligned}$$

$$mc_* = w_*$$

$$\lambda_* = mc_*$$

$$v_*^* = p_*^\theta \frac{z_*}{s_*}$$

$$1 - F(v_*^*) = \frac{(1-(1-\delta)\beta)\, mc_*}{p_* - (1-\delta)\beta mc_*}$$

$$\lambda_* = \frac{\theta - 1}{\theta} \frac{p_*}{\left(\frac{v_*^*}{\Psi\left(p_*^\theta \frac{z_*}{s_*}\right)} + \left(1 - \frac{v_*^*}{\Psi\left(p_*^\theta \frac{z_*}{s_*}\right)}\right)(1-\delta_z)\beta \right)}$$

$$p_*^{1-\theta} \Phi\left(p_*^\theta \frac{z_*}{s_*}\right) = 1$$

Recall

$$\begin{aligned} \Phi(v^*) &= \int_{v \le v^*} v dF(v) + v^{*\frac{\theta-1}{\theta}} \int_{v > v^*} v^{\frac{1}{\theta}} dF(v) \\ &= \exp\left(\frac{\sigma^2}{2}\right) F\left(v^* - \sigma^2\right) + v^{*\frac{\theta-1}{\theta}} \exp\left(\frac{\sigma^2}{2\theta^2}\right) \left(1 - F\left(v^* - \sigma^2/\theta\right)\right) \\ \Psi(v^*) &= \int \min\left(v, v^*\right) dF(v) \\ &= \exp\left(\frac{\sigma^2}{2}\right) F\left(v^* - \sigma^2\right) + v^* \left(1 - F\left(v^*\right)\right) \end{aligned}$$

Log-linearized equations (including aggregation)

$$inv_* \widehat{inv}_t = z_* \widehat{z}_t - s_* \widehat{s}_t - (1-\delta)\left(z_* \widehat{z}_{t-1} - s_* \widehat{s}_{t-1}\right)$$

$$\widehat{c}_t = \widehat{s}_t$$

$$\frac{s_*}{y_*}\widehat{s}_t + \frac{inv_*}{y_*}\widehat{inv}_t = \widehat{y}_t$$

$$\widehat{w}_t = \lambda \widehat{w}_{t-1} - \lambda \widehat{\pi}_t + (1-\lambda)\widehat{c}_t$$

$$\begin{aligned} \widehat{m}_t &= \widehat{c}_t \\ \widehat{\mu}_t &= \widehat{m}_t - \widehat{m}_{t-1} + \widehat{\pi}_t \end{aligned}$$

$$\widehat{\mu}_t = \rho_\mu \widehat{\mu}_{t-1} + \varepsilon_t^\mu$$

$$\widehat{mc}_t = \widehat{w}_t$$

$$\widehat{\lambda}_t = \widehat{mc}_t$$

First derive the equation for I-S ratios:

$$\begin{aligned} \widehat{v}^*_{t+j}(i) &= \theta\left(\widehat{p}_t(i) - \sum_{l=1}^{j}\widehat{\pi}_{t+l}\right) + \widehat{z}_{t+j}(i) - \widehat{s}_{t+j} \\ &= \theta\left(\widehat{\widetilde{p}}_t(i) - \sum_{l=1}^{j}\widehat{\pi}_{t+l}\right) + \widehat{z}_{t+j}(i) - \widehat{s}_{t+j} \end{aligned}$$

Recall I-S equation

$$1 - F(v^*_{t+j}(i)) = \frac{\lambda_{t+j} - (1-\delta) E_{t+j}\left(\frac{\beta c_{t+j}}{c_{t+j+1}} \lambda_{t+j+1}\right)}{p_t(i) \frac{1}{\prod_{l=1}^{j} \pi_{t+l}} - (1-\delta) E_{t+j}\left(\frac{\beta c_{t+j}}{c_{t+j+1}} \lambda_{t+j+1}\right)}$$

log-linearizing gives:

$$\begin{aligned}
& \frac{f(v^*_*) v^*_*}{1 - F(v^*_*)} \left(\widehat{s}_{t+j} - \widehat{z}_{t+j}(i) - \theta \left(\widehat{p}_t(i) - \sum_{l=1}^{j} \widehat{\pi}_{t+l} \right) \right) \\
= \; & \frac{\widehat{\lambda}_{t+j} - (1-\delta)\beta E_{t+j}\left(-\widehat{c}_{t+j+1} + \widehat{c}_{t+j} + \widehat{\lambda}_{t+j+1} \right)}{1 - (1-\delta)\beta} \\
& - \frac{p_* \left(\widehat{p}_t(i) - \sum_{l=1}^{j} \widehat{\pi}_{t+l} \right) - (1-\delta)\beta mc_* E_{t+j}\left(-\widehat{c}_{t+j+1} + \widehat{c}_{t+j} + \widehat{\lambda}_{t+j+1} \right)}{p_* - (1-\delta)\beta mc_*}
\end{aligned}$$

Sum across i and use aggregate price equation

$$\begin{aligned}
& \left\{ \widehat{p}_t(i) + \sum_{j=1}^{\infty} \xi_p^j \left(\widehat{p}_{t-j}(i) - \sum_{l=1}^{j} \widehat{\pi}_{t-l+1} \right) \right\} (1 - \xi_p) \\
= \; & -\frac{p_* \Phi'(v^*_*) v^*_*}{(1-\theta)\,\Phi(v^*_*) + \theta p_* \Phi'(v^*_*) v^*_*} \left(\widehat{z}_t - \widehat{s}_t \right)
\end{aligned}$$

to obtain

$$\begin{aligned}
& \frac{f(v^*_*) v^*_*}{1 - F(v^*_*)} \left(\frac{\theta p_* \Phi'(v^*_*) v^*_*}{(1-\theta)\,\Phi(v^*_*) + \theta p_* \Phi'(v^*_*) v^*_*} - 1 \right) \left(\widehat{z}_t - \widehat{s}_t \right) \\
= \; & \frac{\widehat{\lambda}_t - (1-\delta)\beta E_t \left(-\widehat{c}_{t+1} + \widehat{c}_t + \widehat{\lambda}_{t+1} \right)}{1 - (1-\delta)\beta} \\
& + \frac{p_*}{p_* - (1-\delta)\beta mc_*} \frac{p_* \Phi'(v^*_*) v^*_*}{(1-\theta)\,\Phi(v^*_*) + \theta p_* \Phi'(v^*_*) v^*_*} \left(\widehat{z}_t - \widehat{s}_t \right) \\
& + \frac{(1-\delta)\beta mc_* E_t \left(-\widehat{c}_{t+1} + \widehat{c}_t + \widehat{\lambda}_{t+1} \right)}{p_* - (1-\delta)\beta mc_*}
\end{aligned}$$

Let

$$
\begin{aligned}
B_* &= \frac{p_* \Phi'(v_*^*) v_*^*}{(1-\theta)\,\Phi(v_*^*) + \theta p_* \Phi'(v_*^*) v_*^*} \\
C_* &= \frac{f(v_*^*) v_*^*}{1-F(v_*^*)}\theta - \frac{p_*}{p_* - (1-\delta)\beta mc_*}
\end{aligned}
$$

Simplify by gathering $\widehat{z}_t - \widehat{s}_t$ term:

$$
\begin{aligned}
\left(C_* B_* - \frac{f(v_*^*) v_*^*}{1-F(v_*^*)} \right) (\widehat{z}_t - \widehat{s}_t) &= \frac{\widehat{\lambda}_t - (1-\delta)\beta E_t \left(-\widehat{c}_{t+1} + \widehat{c}_t + \widehat{\lambda}_{t+1} \right)}{1-(1-\delta)\beta} \\
&\quad + \frac{(1-\delta)\beta mc_* E_t \left(-\widehat{c}_{t+1} + \widehat{c}_t + \widehat{\lambda}_{t+1} \right)}{p_* - (1-\delta)\beta mc_*}
\end{aligned}
$$

or after separating markup term from mc-growth term:

$$
\begin{aligned}
&\left(C_* B_* - \frac{f(v_*^*) v_*^*}{1-F(v_*^*)} \right) (\widehat{z}_t - \widehat{s}_t) = \frac{mc_*}{p_* - (1-\delta)\beta mc_*} \widehat{\lambda}_t \\
&+ \frac{p_*/mc_* - 1}{(1-(1-\delta)\beta)\,(p_* - (1-\delta)\beta mc_*)} \left[\widehat{\lambda}_t - (1-\delta)\beta E_t \left(-\widehat{c}_{t+1} + \widehat{c}_t + \widehat{\lambda}_{t+1} \right) \right]
\end{aligned}
$$

Turning to the pricing equation:

$$
\begin{aligned}
&\frac{z_t(i)}{\omega_t(i)} \left[p_t(i) - \frac{\theta}{\theta-1} \left\{ \omega_t(i)\lambda_t + (1-\omega_t(i))(1-\delta) E_t \frac{\beta c_t}{c_{t+1}} \lambda_{t+1} \right\} \right] \\
&+ E_t \sum_{j=0}^{\infty} \xi_p^j \frac{\beta^j c_t}{c_{t+j}} \frac{z_{t+j}(i)}{\omega_{t+j}(i)} \\
&\left[p_t(i) \frac{1}{\prod_{l=1}^{j} \pi_{t+l}} + \frac{\theta}{\theta-1} \left\{ \omega_{t+j}(i)\lambda_{t+j} + (1-\omega_{t+j}(i))(1-\delta) E_{t+j} \frac{\beta c_t}{c_{t+1}} \lambda_{t+j+1} \right\} \right] \\
= \;& 0
\end{aligned}
$$

where $\omega_{t+j}(i) = \frac{v^*_{t+j}(i)}{\int \min(v, v^*_{t+j}(i)) dF(v)}$. Next log-linearize the pricing equation:

$$
\begin{aligned}
& p_* \widehat{p}_t(i) - \frac{\theta}{\theta-1}(1-\delta)\beta mc_* E_t \left[-\widehat{c}_{t+j+1} + \widehat{c}_{t+j} + \widehat{\lambda}_{t+j+1} \right] \\
& - \frac{\theta}{\theta-1}\omega_* \left\{ mc_* \widehat{mc}_t - (1-\delta)\beta mc_* E_t \left[-\widehat{c}_{t+j+1} + \widehat{c}_{t+j} + \widehat{\lambda}_{t+j+1} \right] \right\} \\
& - \frac{\theta}{\theta-1}\omega'_*(v^*_*) v^*_* mc_* (1-(1-\delta)\beta) \left(\theta \widehat{p}_t(i) + \widehat{z}_t(i) - \widehat{s}_t \right) \\
& + E_t \sum_{j=1}^{\infty} \xi_p^j \beta^j \left\{ p_* \widehat{p}_t(i) - \sum_{l=1}^{j} \widehat{\pi}_{t+l} - \frac{\theta}{\theta-1}(1-\delta)\beta mc_* \left[-\widehat{c}_{t+j+1} + \widehat{c}_{t+j} + \widehat{\lambda}_{t+j+1} \right] \right. \\
& - \frac{\theta}{\theta-1}\omega_* \left\{ mc_* \widehat{\lambda}_{t+j} - (1-\delta)\beta mc_* E_t \left[-\widehat{c}_{t+j+1} + \widehat{c}_{t+j} + \widehat{\lambda}_{t+j+1} \right] \right\} \\
& \left. - \frac{\theta}{\theta-1}\omega'_*(v^*_*) v^*_* mc_* (1-(1-\delta)\beta) \left(\theta \left(\widehat{p}_t(i) - \sum_{l=1}^{j} \widehat{\pi}_{t+l} \right) + \widehat{z}_{t+j}(i) - \widehat{s}_{t+j} \right) \right\} \\
= \; & 0
\end{aligned}
$$

where

$$
\begin{aligned}
\Phi'(v^*) &= \frac{\theta-1}{\theta} (v^*)^{\frac{-1}{\theta}} \exp\left(\frac{\sigma^2}{2\theta^2}\right) \left(1 - F\left(v^* - \sigma^2/\theta\right)\right) \\
\Psi(v^*) &= \int \min(v, v^*) \, dF(v) \\
&= \exp\left(\frac{\sigma^2}{2}\right) F\left(v^* - \sigma^2\right) + v^* \left(1 - F(v^*)\right) \\
\Psi'(v^*) &= 1 - F(v^*) \\
\omega(v^*) &= \frac{v^*}{\Psi(v^*)} \\
\omega'(v^*) &= \frac{\Psi(v^*) - v^* \Psi'(v^*)}{\left[\Psi(v^*)\right]^2}
\end{aligned}
$$

minus same at $t+1$ multiplied by $\xi_p\beta$ gives

$$\begin{aligned}
&\widehat{p}_t(i)\left(p_* - \frac{\theta}{\theta-1}\omega'_*(v_*^*)v_*^* mc_*(1-(1-\delta)\beta)\theta\right) - \frac{\theta}{\theta-1}(1-\delta)\beta mc_* E_t\left[-\widehat{c}_{t+1}+\widehat{c}_t+\widehat{\lambda}_{t+1}\right] \\
&-\frac{\theta}{\theta-1}\omega_*\left\{mc_*\widehat{\lambda}_t-(1-\delta)\beta mc_* E_t\left[-\widehat{c}_{t+1}+\widehat{c}_t+\widehat{\lambda}_{t+1}\right]\right\} \\
&-\frac{\theta}{\theta-1}\omega'_*(v_*^*)v_*^* mc_*(1-(1-\delta)\beta)\left(\widehat{z}_t-\widehat{s}_t\right) \\
&-\frac{\xi_p\beta}{1-\xi_p\beta}\left(p_*-\frac{\theta}{\theta-1}\omega'_*(v_*^*)v_*^* mc_*(1-(1-\delta)\beta)\theta\right)E_t\widehat{p}_{t+1}(i) \\
&+E_t\frac{\xi_p\beta}{1-\xi_p\beta}\left(\left(p_*-\frac{\theta}{\theta-1}\omega'_*(v_*^*)v_*^* mc_*(1-(1-\delta)\beta)\theta\right)\widehat{p}_t(i)-\widehat{\pi}_{t+1}\right) \\
=\;& 0
\end{aligned}$$

The aggregate price is determined by

$$\begin{aligned}
1 &= \int_0^1 p_t^{1-\theta}(i)\,\Phi(v_t^*(i))di \\
&= (1-\xi_p)p_t^{1-\theta}(i)\,\Phi(v_t^*(i)) + (1-\xi_p)\sum_{j=1}^{\infty}\xi_p^j p_{t-j}^{1-\theta}(i)\left(\frac{1}{\prod_{l=1}^{j}\pi_{t-l+1}}\right)^{1-\theta}\Phi(v_t^*(i))
\end{aligned}$$

which after log-linearization becomes

$$\begin{aligned}
0 &= (1-\theta)\,\widehat{p}_t(i)\Phi(v_*^*) + p_*^{1-\theta}\Phi'(v_*^*)v_*^*\left(\theta\widehat{p}_t(i)+\widehat{z}_t(i)-\widehat{s}_t\right) \\
&\quad + (1-\theta)\sum_{j=1}^{\infty}\xi_p^j\left(\widehat{p}_{t-j}(i)-\sum_{l=1}^{j}\widehat{\pi}_{t-l+1}\right)\Phi(v_*^*) \\
&\quad + \sum_{j=1}^{\infty}\xi_p^j\left[p_*^{1-\theta}\Phi'(v_*^*)v_*^*\left(\theta\widehat{p}_{t-j}(i)-\theta\sum_{l=1}^{j}\widehat{\pi}_{t-l+1}+\widehat{z}_t(i)-\widehat{s}_t\right)\right]
\end{aligned}$$

$$
\begin{aligned}
0 &= \widehat{p}_t(i) + \frac{1}{1-\xi_p} \frac{p_* \Phi'(v_*^*) v_*^*}{(1-\theta)\,\Phi(v_*^*) + \theta p_* \Phi'(v_*^*) v_*^*} \left(\widehat{z}_t - \widehat{s}_t\right) \\
&\quad + \sum_{j=1}^{\infty} \xi_p^j \left(\widehat{p}_{t-j}(i) - \sum_{l=1}^{j} \widehat{\pi}_{t-l+1} \right)
\end{aligned}
$$

minus same at $t-1$ multiplied by ξ_p :

$$
\widehat{p}_t(i) = \frac{\xi_p}{1-\xi_p} \widehat{\pi}_t - \frac{1}{1-\xi_p} \frac{p_* \Phi'(v_*^*) v_*^*}{(1-\theta)\,\Phi(v_*^*) + \theta p_* \Phi'(v_*^*) v_*^*} \left[\left(\widehat{z}_t - \widehat{s}_t\right) - \xi_p \left(\widehat{z}_{t-1} - \widehat{s}_{t-1}\right) \right]
$$

Let $A_* = p_* - \frac{\theta}{\theta-1} \omega'_*(v_*^*) v_*^* mc_* (1-(1-\delta)\beta)\theta$. Plugging this equation back into the pricing equation yields the NKPC:

$$
\begin{aligned}
& \frac{1}{1-\xi_p\beta} \frac{\xi_p}{1-\xi_p} \widehat{\pi}_t A_* \\
& - \frac{1}{1-\xi_p\beta} \frac{1}{1-\xi_p} B_* \left[\left(\widehat{z}_t - \widehat{s}_t\right) - \xi_p \left(\widehat{z}_{t-1} - \widehat{s}_{t-1}\right) \right] A_* \\
& - \frac{\theta}{\theta-1} (1-\delta) \beta mc_* E_t \left[-\widehat{c}_{t+1} + \widehat{c}_t + \widehat{\lambda}_{t+1} \right] \\
& - \frac{\theta}{\theta-1} \omega_* \left\{ mc_* \widehat{\lambda}_t - (1-\delta)\beta mc_* E_t \left[-\widehat{c}_{t+1} + \widehat{c}_t + \widehat{\lambda}_{t+1} \right] \right\} \\
& - \frac{\theta}{\theta-1} \omega'_*(v_*^*) v_*^* mc_* (1-(1-\delta)\beta) \left(\widehat{z}_t - \widehat{s}_t\right) \\
& - \frac{\xi_p\beta}{1-\xi_p\beta} A_* E_t \left\{ \frac{\xi_p}{1-\xi_p} \widehat{\pi}_{t+1} - \frac{1}{1-\xi_p} B_* \left[\left(\widehat{z}_{t+1} - \widehat{s}_{t+1}\right) - \xi_p \left(\widehat{z}_t - \widehat{s}_t\right) \right] \right\} \\
& - E_t \frac{\xi_p\beta}{1-\xi_p\beta} \widehat{\pi}_{t+1} \\
= \; & 0
\end{aligned}
$$

or after simplification

$$
\begin{aligned}
&\widehat{\pi}_t = \beta \frac{1-\xi_p(1-A_*)}{A_*} E_t \widehat{\pi}_{t+1} \\
&+\frac{\left(1-\xi_p\beta\right)\left(1-\xi_p\right)}{\xi_p A_*}\left[\frac{\theta}{\theta-1}\omega_* mc_* \widehat{\lambda}_t - \left(\omega_*-1\right)\frac{\theta}{\theta-1}(1-\delta)\beta mc_* E_t\left[-\widehat{c}_{t+1}+\widehat{c}_t+\widehat{\lambda}_{t+1}\right]\right] \\
&-B_*\left(\widehat{z}_{t-1}-\widehat{s}_{t-1}\right) \\
&+\left\{\frac{\left(1-\xi_p\beta\right)\left(1-\xi_p\right)}{\xi_p\theta}\frac{p_*-A_*}{A_*}+\frac{B_*}{\xi_p}+\xi_p\beta B_*\right\}\left(\widehat{z}_t-\widehat{s}_t\right) \\
&-\beta B_* E_t\left(\widehat{z}_{t+1}-\widehat{s}_{t+1}\right)
\end{aligned}
$$

Adding adjustment cost

In retailer's problem we add convex adjustment cost in the law of motion for inventory stock:

$$
\begin{aligned}
z_{t+j}(i) &= (1-\delta)\left(z_{t+j-1}(i) - s_{t+j-1}\left(\frac{P_t(i)}{P_{t+j-1}}\right)^{-\theta}\int \min(v, v^*_{t+j}(i))dF(v)\right) + y_{t+j}(i) \\
&\quad -\frac{\eta}{2}\left(y_{t+j}(i) - \left(I_\eta y_{t+j-1}(i) + (1-I_\eta) y_*\right)\right)^2 \left[I_\eta y_{t+j}(i) + (1-I_\eta) y_*\right]^{-1} ,
\end{aligned}
$$

where $I_\eta = \{0,1\}$. The adjustment cost is a convex function of the output level deviations from steady state if $I_\eta = 0$, and if $I_\eta = 1$ the cost a function of the *change* in the output level. The new version of equation for the log-linearized marginal cost of the new stock is

$$
\widehat{\lambda}_{t+j}(i) = \widehat{mc}_{t+j} + \eta\left[\widehat{y}_{t+j}(i) - I_\eta \widehat{y}_{t+j-1}(i)\right] .
$$

Appendix 2: Adding inventories to Smets and Wouters (2007) model

A. New equations

Two equations are added (inventory-sales ratio equation, law of motion for aggregate inventories), and two equations are modified (the pricing equation and the resource constraint).

Resource constraint:

$$\Pi_t = \int \Pi_t(i)\, di = \int P_t(i)\, S_t(i)\, di - W_t L_t - R_t^k K_t$$

taking into account that

$$\int P_t(i)\, S_t(i)\, di = P_t S_t = P_t Y_t - P_t Inv_t$$

where Inv_t is period t inventory investment (change in the end-of-period stock):

$$\begin{aligned} Inv_t &= \int \left[Z_t(i) - S_t(i) - (1-\delta_z)\left(Z_{t-1}(i) - S_{t-1}(i)\right) + \frac{\eta}{2}\left(Y_t(i) - Y_{t-1}(i)\right)^2 Y_t^{-1}(i) \right] di \\ &= Z_t - S_t - (1-\delta_z)\left(Z_{t-1} - S_{t-1}\right) + \frac{\eta}{2} \int \left(Y_t(i) - Y_{t-1}(i)\right)^2 Y_t^{-1}(i)\, di \end{aligned}$$

Hence the resource constraint is

$$C_t + I_t + Inv_t + G_t + a(u_t)\overline{K}_{t-1} = Y_t$$

and final sales to domestic purchasers are defined as

$$C_t + I_t + G_t + a(u_t)\overline{K}_{t-1} = S_t$$

Final good producers:

$$\begin{aligned}
&\max_{S_t, S_t(i,v)} \; P_t S_t - \int_0^1 P_t(i)\, S_t(i,v)\, di \\
&s.t. \qquad S_t = \left[\int_0^1 \left(v_t(i)^{\frac{1}{\theta_t}} S_t(i,v)^{\frac{\theta_t - 1}{\theta_t}} \right) di \right]^{\frac{\theta_t}{\theta_t - 1}} \\
&\qquad\qquad S_t(i,v) \leq Z_t(i) \qquad\qquad\qquad\qquad [\widetilde{\gamma}_t(i,v)]
\end{aligned}$$

FOCs:

$$\begin{aligned}
P_t v_t(i)^{\frac{1}{\theta_t}} S_t(i,v)^{\frac{-1}{\theta_t}} S_t^{\frac{1}{\theta_t}} - P_t(i) - \widetilde{\gamma}_t(i,v) &= 0 \qquad (1) \\
\widetilde{\gamma}_t(i,v)\left(Z_t(i) - S_t(i,v)\right) &= 0
\end{aligned}$$

Case 1: $\widetilde{\gamma}_t(i,v) > 0$, then $S_t(i,v) = Z_t(i)$, and $\widetilde{\gamma}_t(i,v) = P_t v_t(i)^{\frac{1}{\theta_t}} Z_t(i)^{\frac{-1}{\theta_t}} S_t^{\frac{1}{\theta_t}} - P_t(i)$

Case 2: $\widetilde{\gamma}_t(i,v) = 0$, then $S_t(i,v) = v_t(i) S_t \left(\frac{P_t(i)}{P_t}\right)^{-\theta_t}$.

Intermediate good producers:

$$\max_{\widetilde{P}_t(i),\{Z_{t+j}(i)\}} \quad E_t \sum_{j=0}^{\infty} \xi_p^j \frac{\beta^j \Xi_{t+j} P_t}{\Xi_t P_{t+j}} \times$$

$$\left[\widetilde{P}_t(i) X_{t,j} S_{t+j} \left(\frac{\widetilde{P}_t\left(i\right) X_{t,j}}{P_{t+j}} \right)^{-\theta_{t+j}} \int \min(v, v_{t+j}^*(i)) dF(v) - MC_{t+j} Y_{t+j}\left(i\right) \right]$$

$$+ E_t \sum_{j=1}^{\infty} \xi_p^{j-1}(1-\xi_p) \frac{\beta^j \Xi_{t+j} P_t}{\Xi_t P_{t+j}} \left[-MC_{t+j} Y_{t+j}\left(i\right) \right]$$

$$s.t. \quad v_{t+j}^*(i) = \frac{Z_{t+j}\left(i\right)}{S_{t+j}} \left(\frac{\widetilde{P}_t\left(i\right) X_{t,j}}{P_{t+j}} \right)^{\theta_{t+j}}$$

$$Z_{t+j}\left(i\right) = (1-\delta_z) \left(Z_{t+j-1}\left(i\right) - S_{t+j-1} \left(\frac{\widetilde{P}_t\left(i\right) X_{t,j-1}}{P_{t+j-1}} \right)^{-\theta_{t+j-1}} \int \min(v, v_{t+j-1}^*(i)) dF(v) \right) +$$

$$-\frac{\eta}{2} \left(Y_{t+j}\left(i\right) - \gamma Y_{t+j-1}\left(i\right) \right)^2 Y_{t+j}^{-1}\left(i\right)$$

Using the fact that

$$\frac{d}{dz} \int \min(v, v^*(z)) dF(v) = (1 - F(v^*(z))) \frac{dv^*(z)}{dz}$$

$$[Y]: \quad -MC_{t+j} + \lambda_{t+j}\left(i\right) \left(1 - \eta \left(1 - \gamma \frac{Y_{t+j-1}\left(i\right)}{Y_{t+j}\left(i\right)} \right) + \frac{\eta}{2} \left(1 - \gamma \frac{Y_{t+j-1}\left(i\right)}{Y_{t+j}\left(i\right)} \right)^2 \right) = 0$$

So that the shadow value of stock $\lambda_{t+j}\left(i\right) = MC_{t+j} / \left[1 - \eta \left(1 - \gamma \frac{Y_{t+j-1}(i)}{Y_{t+j}(i)} \right) + \frac{\eta}{2} \left(1 - \gamma \frac{Y_{t+j-1}(i)}{Y_{t+j}(i)} \right)^2 \right]$.

FOCs for $Z_{t+j}\left(i\right)$

$$1 - F(v_{t+j}^*(i)) = \frac{\lambda_{t+j}\left(i\right) - (1-\delta_z) E_{t+j} \frac{\beta \Xi_{t+j+1} P_{t+j}}{\Xi_{t+j} P_{t+j+1}} \lambda_{t+j+1}\left(i\right)}{\widetilde{P}_t(i) X_{t,j} - (1-\delta_z) E_{t+j} \frac{\beta \Xi_{t+j+1} P_{t+j}}{\Xi_{t+j} P_{t+j+1}} \lambda_{t+j+1}\left(i\right)}$$

FOCs for $\widetilde{P}_t(i)$:

$$
\begin{aligned}
& E_t \sum_{j=0}^{\infty} \xi_p^j \frac{\beta^j \Xi_{t+j} P_t}{\Xi_t P_{t+j}} \times \\
& \left[\begin{array}{c}
(1-\theta_t)\widetilde{P}_t(i) X_{t,j} S_{t+j} \left(\frac{\widetilde{P}_t(i)X_{t,j}}{P_{t+j}}\right)^{-\theta_{t+j}} \int \min(v, v^*_{t+j}(i)) dF(v) + \theta_{t+j}\widetilde{P}_t(i) X_{t,j} Z_{t+j}(i)(1-F(v^*_{t+j}(\\
+(1-\delta_z) E_{t+j} \frac{\beta \Xi_{t+j+1} P_{t+j}}{\Xi_{t+j} P_{t+j+1}} \lambda_{t+j+1}(i) \times \\
\left\{ \theta_{t+j} S_{t+j} \left(\frac{\widetilde{P}_t(i)X_{t,j}}{P_{t+j}}\right)^{-\theta_{t+j}} \int \min(v, v^*_{t+j}(i)) dF(v) - \theta_{t+j} Z_{t+j}(i)(1-F(v^*_{t+j}(i))) \right\}
\end{array} \right. \\
= \; & E_t \sum_{j=0}^{\infty} \xi_p^j \frac{\beta^j \Xi_{t+j} P_t}{\Xi_t P_{t+j}} \frac{Z_{t+j}(i)}{v^*_{t+j}(i)} \times \\
& \left[\begin{array}{c}
(1-\theta_{t+j})\widetilde{P}_t(i) X_{t,j} \int \min(v, v^*_{t+j}(i)) dF(v) + \theta_{t+j}\widetilde{P}_t(i) X_{t,j} v^*_{t+j}(i)(1-F(v^*_{t+j}(i))) \\
+(1-\delta_z) E_{t+j} \frac{\beta \Xi_{t+j+1} P_{t+j}}{\Xi_{t+j} P_{t+j+1}} \lambda_{t+j+1}(i) \times \\
\left\{ \theta_{t+j} \int \min(v, v^*_{t+j}(i)) dF(v) - \theta_{t+j} v^*_{t+j}(i)(1-F(v^*_{t+j}(i))) \right\}
\end{array} \right] \\
= \; & E_t \sum_{j=0}^{\infty} \xi_p^j \frac{\beta^j \Xi_{t+j} P_t}{\Xi_t P_{t+j}} Z_{t+j}(i) \frac{\int \min(v, v^*_{t+j}(i)) dF(v)}{v^*_{t+j}(i)} \times \\
& \left[\begin{array}{c}
(1-\theta_{t+j})\widetilde{P}_t(i) X_{t,j} + \theta_{t+j}(1-\delta_z) E_{t+j} \frac{\beta \Xi_{t+j+1} P_{t+j}}{\Xi_{t+j} P_{t+j+1}} \lambda_{t+j+1}(i) \\
+\theta_{t+j} \left\{ \lambda_{t+j}(i) - (1-\delta_z) E_{t+j} \frac{\beta \Xi_{t+j+1} P_{t+j}}{\Xi_{t+j} P_{t+j+1}} \lambda_{t+j+1}(i) \right\} \frac{v^*_{t+j}(i)}{\int \min(v, v^*_{t+j}(i)) dF(v)}
\end{array} \right]
\end{aligned}
$$

For the aggregate price we obtain

$$
\begin{aligned}
P_t^{1-\theta} &= \left[\int_{v_t(i) \le v^*_t(i)} v_t(i) P_t(i)^{1-\theta} \, di + \int_{v_t(i) > v^*_t(i)} v_t(i) \left(\frac{v^*_t(i)}{v_t(i)}\right)^{\frac{\theta-1}{\theta}} P_t(i)^{1-\theta} \, di \right] \\
&= \int_0^1 P_t^{1-\theta}(i) \, \Phi(v^*_t(i)) di
\end{aligned}
$$

where we invoke the law of large numbers and the fact that $v_t(i)$ is independent of $P_t(i)$ and

$z_t(i)$. Here $\Phi(v_t^*(i)$ satisfies:

$$\Phi(v_t^*(i)) = \int_{v_t(i)\leq v_t^*(i)} v_t(i) dF(v) + v_t^*(i)^{\frac{\theta-1}{\theta}} \int_{v_t(i)>v_t^*(i)} v_t(i)^{\frac{1}{\theta}} dF(v)$$

For normally distributed demand shocks

$$\Phi(v_t^*(i)) = \exp\left(\frac{\sigma^2}{2}\right) F\left(v_t^*(i) - \sigma^2\right) + v_t^*(i)^{\frac{\theta-1}{\theta}} \exp\left(\frac{\sigma^2}{2\theta^2}\right)\left(1 - F\left(v_t^*(i) - \sigma^2/\theta\right)\right)$$

which implies

$$\begin{aligned}
\Phi'(v^*) &= \frac{\theta-1}{\theta}\left(v^*\right)^{\frac{-1}{\theta}} \exp\left(\frac{\sigma^2}{2\theta^2}\right)\left(1 - F\left(v^* - \sigma^2/\theta\right)\right) \\
\Psi(v^*) &= \int \min\left(v, v_t^*(i)\right) dF(v) \\
&= \exp\left(\frac{\sigma^2}{2}\right) F\left(v_t^*(i) - \sigma^2\right) + v_t^*(i)\left(1 - F\left(v_t^*(i)\right)\right) \\
\Psi'(v^*) &= 1 - F\left(v_t^*(i)\right) \\
\omega(v^*) &= \frac{v^*}{\Psi(v^*)} \\
\omega'(v^*) &= \frac{\Psi(v^*) - v^*\Psi'(v^*)}{\left[\Psi(v^*)\right]^2}
\end{aligned}$$

Stationary equations

$$inv_t = z_t - s_t - \frac{1-\delta_z}{\gamma}\left(z_{t-1} - s_{t-1}\right) + \frac{\eta}{2}\int \left(y_t(i) - y_{t-1}(i)\right)^2 y_t^{-1}(i) di$$

$$c_t + i_t + y_* g_t + \frac{a(u_t)}{\gamma}\overline{k}_{t-1} = s_t$$

$$s_t + inv_t = y_t$$

$$v^*_{t+j}(i) = p_t^{\theta_{t+j}}(i)\,\frac{z_{t+j}(i)}{s_{t+j}}$$

$$\lambda_{t+j}(i) = mc_{t+j}/\left[1-\eta\left(1-\frac{y_{t+j-1}(i)}{y_{t+j}(i)}\right)+\frac{\eta}{2}\left(1-\frac{y_{t+j-1}(i)}{y_{t+j}(i)}\right)^2\right]$$

$$1-F(v^*_{t+j}(i)) = \frac{\lambda_{t+j}-(1-\delta_z)E_{t+j}\left(\frac{\overline{\beta}\gamma\xi_{t+j+1}}{\xi_{t+j}}\lambda_{t+j+1}\right)}{\widetilde{p}_t(i)\frac{\prod_{l=1}^{j}\pi_{t+l-1}^{\iota_p}\pi_*^{1-\iota_p}}{\prod_{l=1}^{j}\pi_{t+l}}-(1-\delta_z)E_{t+j}\left(\frac{\overline{\beta}\gamma\xi_{t+j+1}}{\xi_{t+j}}\lambda_{t+j+1}\right)}$$

$$\begin{aligned}
&\frac{z_t(i)}{\omega_t(i)}(\theta_t-1)\left[\widetilde{p}_t(i)-\frac{\theta_t}{\theta_t-1}\left\{\omega_t(i)\lambda_t+(1-\omega_t(i))(1-\delta_z)E_t\frac{\overline{\beta}\gamma\xi_{t+1}}{\xi_t}\lambda_{t+1}\right\}\right]\\
&+E_t\sum_{j=1}^{\infty}\xi_p^j\frac{\overline{\beta}^j\gamma^j\xi_{t+j}}{\xi_t}\frac{z_{t+j}(i)}{\omega_{t+j}(i)}(\theta_{t+j}-1)\times\\
&\times\left[\widetilde{p}_t(i)\frac{\prod_{l=1}^{j}\pi_{t+l-1}^{\iota_p}\pi_*^{1-\iota_p}}{\prod_{l=1}^{j}\pi_{t+l}}-\frac{\theta_{t+j}}{\theta_{t+j}-1}\left\{\omega_{t+j}(i)\lambda_{t+j}+(1-\omega_{t+j}(i))(1-\delta_z)E_{t+j}\frac{\overline{\beta}\gamma\xi_{t+j+1}}{\xi_{t+j}}\lambda_{t+j+1}\right\}\right]\\
=\;&0
\end{aligned}$$

where $\omega_{t+j}(i) = \frac{v^*_{t+j}(i)}{\int \min(v, v^*_{t+j}(i))dF(v)}$.

$$\begin{aligned}
1 &= \int_0^1 p_t^{1-\theta_t}(i)\,\Phi(v_t^*(i))di\\
&= (1-\xi_p)\widetilde{p}_t^{1-\theta_t}(i)\,\Phi(v_t^*(i)) + (1-\xi_p)\sum_{j=1}^{\infty}\xi_p^j\widetilde{p}_{t-j}^{1-\theta_t}(i)\left(\frac{\prod_{l=1}^{j}\pi_{t-l}^{\iota_p}\pi_*^{1-\iota_p}}{\prod_{l=1}^{j}\pi_{t-l+1}}\right)^{1-\theta_t}\Phi(v_t^*(i))
\end{aligned}$$

B. Steady state

$$inv_* = \left(1-\frac{1-\delta_z}{\gamma}\right)(z_*-s_*)$$

$$c_* + i_* + y_* g_* = s_*$$

$$s_* + inv_* = y_*$$

$$v_*^* = p_*^\theta \frac{z_*}{s_*}$$

$$\lambda_* = mc_*$$

$$1 - F(v_*^*) = \frac{\left(1 - (1-\delta_z)\overline{\beta}\gamma\right)\lambda_*}{\widetilde{p}_* - (1-\delta_z)\overline{\beta}\gamma\lambda_*}$$

$$\widetilde{p}_*^{1-\theta} \Phi\left(p_*^\theta \frac{z_*}{s_*}\right) = 1$$

$$\begin{aligned} & \widetilde{p}_* - \frac{\theta}{\theta-1}\left(\frac{v_*^*}{\Psi\left(p_*^\theta \frac{z_*}{s_*}\right)}\lambda_* + \left(1 - \frac{v_*^*}{\Psi\left(p_*^\theta \frac{z_*}{s_*}\right)}\right)(1-\delta_z)\overline{\beta}\gamma\lambda_*\right) \\ = \ & \widetilde{p}_* - \frac{\theta}{\theta-1}(1-\delta_z)\overline{\beta}\gamma\lambda_* - \frac{\theta}{\theta-1}\frac{v_*^*}{\Psi\left(p_*^\theta \frac{z_*}{s_*}\right)}\lambda_*\left(1 - (1-\delta_z)\overline{\beta}\gamma\right) = 0 \end{aligned}$$

or

$$\lambda_* = \frac{\theta-1}{\theta}\frac{\widetilde{p}_*}{\left(\frac{v_*^*}{\Psi\left(p_*^\theta \frac{z_*}{s_*}\right)} + \left(1 - \frac{v_*^*}{\Psi\left(p_*^\theta \frac{z_*}{s_*}\right)}\right)(1-\delta_z)\overline{\beta}\gamma\right)}$$

Log-linearized equations

$$inv_* \widehat{inv}_t = z_* \widehat{z}_t - s_* \widehat{s}_t - \frac{1-\delta_z}{\gamma}\left(z_* \widehat{z}_{t-1} - s_* \widehat{s}_{t-1}\right)$$

$$\frac{c_*}{y_*}\widehat{c}_t + \frac{i_*}{y_*}\widehat{i}_t + \widehat{g}_t + \frac{r_*^k k_*}{y_*}\widehat{u}_t = \frac{s_*}{y_*}\widehat{s}_t$$

$$\frac{s_*}{y_*}\widehat{s}_t + \frac{inv_*}{y_*}\widehat{inv}_t = \widehat{y}_t$$

First derive the equation for I-S ratios:

$$\begin{aligned}
\widehat{v}^*_{t+j}(i) &= \theta\left(\widehat{\widetilde{p}}_t(i) + \sum_{l=1}^{j}\left[\iota_p\widehat{\pi}_{t+l-1} - \widehat{\pi}_{t+l}\right]\right) + \widehat{z}_{t+j}(i) - \widehat{s}_{t+j} + \widehat{\theta}_{t+j}\ln\widetilde{p}^{\theta}_* \\
&= \theta\left(\widehat{\widetilde{p}}_t(i) + \sum_{l=1}^{j}\left[\iota_p\widehat{\pi}_{t+l-1} - \widehat{\pi}_{t+l}\right]\right) + \widehat{z}_{t+j}(i) - \widehat{s}_{t+j} - \widehat{\lambda}_{p,t+j}\ln\widetilde{p}^{\theta-1}_*
\end{aligned}$$

$$\widehat{\lambda}_{t+j}\left(i\right) = \widehat{mc}_{t+j} + \eta\left[\widehat{y}_{t+j}\left(i\right) - \widehat{y}_{t+j-1}\left(i\right)\right]$$

Recall I-S equation:

$$1 - F(v^*_{t+j}(i)) = \frac{mc_{t+j} - (1-\delta_z)E_{t+j}\left(\frac{\overline{\beta}\gamma\xi_{t+j+1}}{\xi_{t+j}}mc_{t+j+1}\right)}{\widetilde{p}_t(i)\frac{\prod_{l=1}^{j}\pi^{\iota_p}_{t+l-1}\pi^{1-\iota_p}_*}{\prod_{l=1}^{j}\pi_{t+l}} - (1-\delta_z)E_{t+j}\left(\frac{\overline{\beta}\gamma\xi_{t+j+1}}{\xi_{t+j}}mc_{t+j+1}\right)}$$

which after log-linearization becomes:

$$\begin{aligned}
&\frac{f(v^*_*)v^*_*}{1-F(v^*_*)}\left(\widehat{s}_{t+j} - \widehat{z}_{t+j}(i) - \theta\left(\widehat{\widetilde{p}}_t(i) + \sum_{l=1}^{j}\left[\iota_p\widehat{\pi}_{t+l-1} - \widehat{\pi}_{t+l}\right]\right) + \widehat{\lambda}_{p,t+j}\ln\widetilde{p}^{\theta-1}_*\right) \\
=\ &\frac{\widehat{\lambda}_{t+j} - (1-\delta_z)\overline{\beta}\gamma E_{t+j}\left(\widehat{\xi}_{t+j+1} - \widehat{\xi}_{t+j} + \widehat{\lambda}_{t+j+1}\right)}{1-(1-\delta_z)\overline{\beta}\gamma} \\
&-\frac{\widetilde{p}_*\left(\widehat{\widetilde{p}}_t(i) + \sum_{l=1}^{j}\left[\iota_p\widehat{\pi}_{t+l-1} - \widehat{\pi}_{t+l}\right]\right) - (1-\delta_z)\overline{\beta}\gamma mc_* E_{t+j}\left(\widehat{\xi}_{t+j+1} - \widehat{\xi}_{t+j} + \widehat{\lambda}_{t+j+1}\right)}{\widetilde{p}_* - (1-\delta_z)\overline{\beta}\gamma mc_*}
\end{aligned}$$

Aggregate and use aggregate price equation

$$\begin{aligned}
&\left\{\widehat{\widetilde{p}}_t(i) + \sum_{j=1}^{\infty} \xi_p^j \left(\widehat{\widetilde{p}}_{t-j}(i) + \sum_{l=1}^{j} \left[\iota_p \widehat{\pi}_{t-l} - \widehat{\pi}_{t-l+1}\right]\right)\right\}(1-\xi_p) \\
= \;& \widehat{\lambda}_{p,t} \ln \widetilde{p}_* - \frac{\widetilde{p}_* \Phi'(v_*^*) v_*^*}{(1-\theta)\,\Phi(v_*^*) + \theta \widetilde{p}_* \Phi'(v_*^*) v_*^*} \left(\widehat{z}_t - \widehat{s}_t - \widehat{\lambda}_{p,t} \ln \widetilde{p}_*^{\theta-1}\right)
\end{aligned}$$

to obtain

$$\begin{aligned}
&\frac{f(v_*^*) v_*^*}{1-F(v_*^*)} \left(-\theta \widehat{\lambda}_{p,t} \ln \widetilde{p}_* + \left(\frac{\theta \widetilde{p}_* \Phi'(v_*^*) v_*^*}{(1-\theta)\,\Phi(v_*^*) + \theta \widetilde{p}_* \Phi'(v_*^*) v_*^*} - 1\right) \left(\widehat{z}_t - \widehat{s}_t - \widehat{\lambda}_{p,t} \ln \widetilde{p}_*^{\theta-1}\right)\right) \\
= \;& \frac{\widehat{mc}_t - (1-\delta_z)\overline{\beta}\gamma E_t \left(\widehat{\xi}_{t+1} - \widehat{\xi}_t + \widehat{\lambda}_{t+1}\right)}{1-(1-\delta_z)\overline{\beta}\gamma} \\
&- \frac{\widetilde{p}_* \left(\widehat{\lambda}_{p,t} \ln \widetilde{p}_* - \frac{\widetilde{p}_* \Phi'(v_*^*) v_*^*}{(1-\theta)\Phi(v_*^*) + \theta \widetilde{p}_* \Phi'(v_*^*) v_*^*} \left(\widehat{z}_t - \widehat{s}_t - \widehat{\lambda}_{p,t} \ln \widetilde{p}_*^{\theta-1}\right)\right)}{\widetilde{p}_* - (1-\delta_z)\overline{\beta}\gamma mc_*} \\
&+ \frac{(1-\delta_z)\overline{\beta}\gamma mc_* E_t \left(\widehat{\xi}_{t+1} - \widehat{\xi}_t + \widehat{\lambda}_{t+1}\right)}{\widetilde{p}_* - (1-\delta_z)\overline{\beta}\gamma mc_*}
\end{aligned}$$

Let $B_* = \frac{\widetilde{p}_* \Phi'(v_*^*) v_*^*}{(1-\theta)\Phi(v_*^*) + \theta \widetilde{p}_* \Phi'(v_*^*) v_*^*}$, $C_* = \frac{f(v_*^*) v_*^*}{1-F(v_*^*)}\theta - \frac{\widetilde{p}_*}{\widetilde{p}_* - (1-\delta_z)\overline{\beta}\gamma mc_*}$. Simplify by gathering $\widehat{z}_t - \widehat{s}_t - \widehat{\lambda}_{p,t} \ln \widetilde{p}_*^{\theta-1}$ term:

$$\begin{aligned}
&\left(C_* B_* - \frac{f(v_*^*) v_*^*}{1-F(v_*^*)}\right) \left(\widehat{z}_t - \widehat{s}_t - \widehat{\lambda}_{p,t} \ln \widetilde{p}_*^{\theta-1}\right) = C_* \widehat{\lambda}_{p,t} \ln \widetilde{p}_* \\
&+ \frac{\widehat{mc}_t - (1-\delta_z)\overline{\beta}\gamma E_t \left(\widehat{\xi}_{t+1} - \widehat{\xi}_t + \widehat{\lambda}_{t+1}\right)}{1-(1-\delta_z)\overline{\beta}\gamma} + \frac{(1-\delta_z)\overline{\beta}\gamma mc_* E_t \left(\widehat{\xi}_{t+1} - \widehat{\xi}_t + \widehat{\lambda}_{t+1}\right)}{\widetilde{p}_* - (1-\delta_z)\overline{\beta}\gamma mc_*}
\end{aligned}$$

Turning to the pricing equation:

$$
\begin{aligned}
& \frac{z_t(i)}{\omega_t(i)}\left[\widetilde{p}_t(i) - \frac{\theta_t}{\theta_t - 1}\left\{\omega_t(i) mc_t + (1-\omega_t(i))(1-\delta_z)E_t \frac{\overline{\beta}\gamma\xi_{t+1}}{\xi_t} mc_{t+1}\right\}\right] \\
& + E_t \sum_{j=0}^{\infty} \xi_p^j \frac{\overline{\beta}^j \gamma^j \xi_{t+j}}{\xi_t} \frac{z_{t+j}(i)}{\omega_{t+j}(i)} \\
& \left[\widetilde{p}_t(i) \frac{\prod_{l=1}^{j} \pi_{t+l-1}^{\iota_p} \pi_*^{1-\iota_p}}{\prod_{l=1}^{j} \pi_{t+l}} + \frac{\theta_t}{\theta_t - 1}\left\{\omega_{t+j}(i) mc_{t+j} + (1-\omega_{t+j}(i))(1-\delta_z)E_{t+j} \frac{\overline{\beta}\gamma\xi_{t+j+1}}{\xi_{t+j}} mc_{t+j+1}\right\}\right] \\
= \; & 0
\end{aligned}
$$

where $\omega_{t+j}(i) = \frac{v_{t+j}^*(i)}{\int \min(v, v_{t+j}^*(i)) dF(v)}$.Next log-linearize the pricing equation:

$$
\begin{aligned}
& \widetilde{p}_* \widehat{\widetilde{p}}_t(i) - \frac{\theta}{\theta - 1}(1-\delta_z)\overline{\beta}\gamma mc_* E_t\left[\widehat{\xi}_{t+1} - \widehat{\xi}_t + \widehat{\lambda}_{t+1}\right] \\
& -\frac{\theta}{\theta-1}\omega_*\left\{mc_*\widehat{\lambda}_t - (1-\delta_z)\overline{\beta}\gamma mc_* E_t\left[\widehat{\xi}_{t+1} - \widehat{\xi}_t + \widehat{\lambda}_{t+1}\right]\right\} \\
& -\frac{\theta}{\theta-1}\omega_*'(v_*^*)v_*^* mc_*(1-(1-\delta_z)\overline{\beta}\gamma)\left(\theta\widehat{\widetilde{p}}_t(i) + \widehat{z}_t(i) - \widehat{s}_t - \widehat{\lambda}_{p,t}\ln\widetilde{p}_*^{\theta-1}\right) \\
& -\frac{\theta}{\theta-1}\left\{\omega_* mc_* + (1-\omega_*)(1-\delta_z)\overline{\beta}\gamma mc_*\right\}\frac{1}{\theta}\widehat{\lambda}_{p,t} \\
& +E_t\sum_{j=1}^{\infty}\xi_p^j\overline{\beta}^j\gamma^j\left\{\widetilde{p}_*\widehat{\widetilde{p}}_t(i) + \sum_{l=1}^{j}\left[\iota_p\widehat{\pi}_{t+l-1} - \widehat{\pi}_{t+l}\right] - \frac{\theta}{\theta-1}(1-\delta_z)\overline{\beta}\gamma mc_*\left[\widehat{\xi}_{t+j+1} - \widehat{\xi}_{t+j} + \widehat{\lambda}_{t+j+1}\right]\right. \\
& -\frac{\theta}{\theta-1}\omega_*\left\{mc_*\widehat{\lambda}_{t+j} - (1-\delta_z)\overline{\beta}\gamma mc_* E_t\left[\widehat{\xi}_{t+j+1} - \widehat{\xi}_{t+j} + \widehat{\lambda}_{t+j+1}\right]\right\} \\
& -\frac{\theta}{\theta-1}\left\{\omega_* mc_* + (1-\omega_*)(1-\delta_z)\overline{\beta}\gamma mc_*\right\}\frac{1}{\theta}\widehat{\lambda}_{p,t+j} \\
& -\frac{\theta}{\theta-1}\omega_*'(v_*^*)v_*^* mc_*(1-(1-\delta_z)\overline{\beta}\gamma)\times \\
& \qquad \left.\times\left(\theta\left(\widehat{\widetilde{p}}_t(i) + \sum_{l=1}^{j}\left[\iota_p\widehat{\pi}_{t+l-1} - \widehat{\pi}_{t+l}\right]\right) + \widehat{z}_{t+j}(i) - \widehat{s}_{t+j} - \widehat{\lambda}_{p,t+j}\ln\widetilde{p}_*^{\theta-1}\right)\right\} \\
= \; & 0
\end{aligned}
$$

where $\omega_* = \frac{v_*^*}{\Psi(v_*^*)}$, $\omega_*' = \frac{\Psi(v_*^*) - v_*^*\Psi'(v_*^*)}{[\Psi(v_*^*)]^2}$, minus same at $t+1$ multiplied by $\xi_p\overline{\beta}\gamma$ gives

$$\begin{aligned}
&\widehat{\widetilde{p}}_t(i)\left(\widetilde{p}_* - \frac{\theta}{\theta-1}\omega_*'(v_*^*)v_*^* mc_*(1-(1-\delta_z)\overline{\beta}\gamma)\theta\right) - \frac{\theta}{\theta-1}(1-\delta_z)\overline{\beta}\gamma mc_* E_t\left[\widehat{\xi}_{t+1} - \widehat{\xi}_t + \widehat{\lambda}_{t+1}\right] \\
&-\frac{\theta}{\theta-1}\omega_*\left\{mc_*\widehat{\lambda}_t - (1-\delta_z)\overline{\beta}\gamma mc_* E_t\left[\widehat{\xi}_{t+1} - \widehat{\xi}_t + \widehat{\lambda}_{t+1}\right]\right\} \\
&-\frac{\theta}{\theta-1}\omega_*'(v_*^*)v_*^* mc_*(1-(1-\delta_z)\overline{\beta}\gamma)\left(\widehat{z}_t - \widehat{s}_t - \widehat{\lambda}_{p,t}\ln\widetilde{p}_*^{\theta-1}\right) \\
&-\frac{\theta}{\theta-1}\left\{\omega_* mc_* + (1-\omega_*)(1-\delta_z)\overline{\beta}\gamma mc_*\right\}\frac{1}{\theta}\widehat{\lambda}_{p,t} \\
&-\frac{\xi_p\overline{\beta}\gamma}{1-\xi_p\overline{\beta}\gamma}\left(\widetilde{p}_* - \frac{\theta}{\theta-1}\omega_*'(v_*^*)v_*^* mc_*(1-(1-\delta_z)\overline{\beta}\gamma)\theta\right)E_t\widehat{\widetilde{p}}_{t+1}(i) \\
&+E_t\frac{\xi_p\overline{\beta}\gamma}{1-\xi_p\overline{\beta}\gamma}\left(\left(\widetilde{p}_* - \frac{\theta}{\theta-1}\omega_*'(v_*^*)v_*^* mc_*(1-(1-\delta_z)\overline{\beta}\gamma)\theta\right)\widehat{\widetilde{p}}_t(i) + \left[\iota_p\widehat{\pi}_t - \widehat{\pi}_{t+1}\right]\right) \\
=\;& 0
\end{aligned}$$

The aggregate price is determined by

$$\begin{aligned}
1 &= \int_0^1 p_t^{1-\theta}(i)\,\Phi(v_t^*(i))di \\
&= (1-\xi_p)\widetilde{p}_t^{1-\theta}(i)\,\Phi(v_t^*(i)) + (1-\xi_p)\sum_{j=1}^{\infty}\xi_p^j\widetilde{p}_{t-j}^{1-\theta}(i)\left(\frac{\prod_{l=1}^{j}\pi_{t-l}^{\iota_p}\pi_*^{1-\iota_p}}{\prod_{l=1}^{j}\pi_{t-l+1}}\right)^{1-\theta}\Phi(v_t^*(i))
\end{aligned}$$

and after log-linearization it becomes:

$$\begin{aligned}
0 &= (1-\theta)\left[\widehat{\widetilde{p}}_t(i) - \widehat{\lambda}_{p,t}\ln\widetilde{p}_*\right]\Phi(v_*^*) + \widetilde{p}_*^{1-\theta}\Phi'(v_*^*)v_*^*\left(\theta\widehat{\widetilde{p}}_t(i) + \widehat{z}_t(i) - \widehat{s}_t - \widehat{\lambda}_{p,t}\ln\widetilde{p}_*^{\theta-1}\right) \\
&\quad+(1-\theta)\sum_{j=1}^{\infty}\xi_p^j\left(\widehat{\widetilde{p}}_{t-j}(i) + \sum_{l=1}^{j}\left[\iota_p\widehat{\pi}_{t-l} - \widehat{\pi}_{t-l+1}\right] - \widehat{\lambda}_{p,t}\ln\widetilde{p}_*\right)\Phi(v_*^*) \\
&\quad+\sum_{j=1}^{\infty}\xi_p^j\left[\widetilde{p}_*^{1-\theta}\Phi'(v_*^*)v_*^*\left(\theta\widehat{\widetilde{p}}_{t-j}(i) + \theta\sum_{l=1}^{j}\left[\iota_p\widehat{\pi}_{t-l} - \widehat{\pi}_{t-l+1}\right] + \widehat{z}_t(i) - \widehat{s}_t - \widehat{\lambda}_{p,t}\ln\widetilde{p}_*^{\theta-1}\right)\right]
\end{aligned}$$

or after simplication,

$$
\begin{aligned}
0 \;=\; & \widehat{\widetilde{p}}_t(i) - \widehat{\lambda}_{p,t} \ln \widetilde{p}_* + \frac{1}{1-\xi_p} \frac{\widetilde{p}_* \Phi'(v_*^*) v_*^*}{(1-\theta)\,\Phi(v_*^*) + \theta \widetilde{p}_* \Phi'(v_*^*) v_*^*} \left(\widehat{z}_t - \widehat{s}_t - \widehat{\lambda}_{p,t} \ln \widetilde{p}_*^{\theta-1} \right) \\
& + \sum_{j=1}^{\infty} \xi_p^j \left(\widehat{\widetilde{p}}_{t-j}(i) + \sum_{l=1}^{j} \left[\iota_p \widehat{\pi}_{t-l} - \widehat{\pi}_{t-l+1} \right] - \widehat{\lambda}_{p,t} \ln \widetilde{p}_* \right)
\end{aligned}
$$

minus same at $t-1$ multiplied by ξ_p :

$$
\begin{aligned}
& \widehat{\widetilde{p}}_t(i) - \frac{1}{1-\xi_p} \widehat{\lambda}_{p,t} \ln \widetilde{p}_* = \frac{\xi_p}{1-\xi_p} \left[\widehat{\pi}_t - \iota_p \widehat{\pi}_{t-1} \right] \\
& - \frac{1}{1-\xi_p} \frac{\widetilde{p}_* \Phi'(v_*^*) v_*^*}{(1-\theta)\,\Phi(v_*^*) + \theta \widetilde{p}_* \Phi'(v_*^*) v_*^*} \left[\left(\widehat{z}_t - \widehat{s}_t - \widehat{\lambda}_{p,t} \ln \widetilde{p}_*^{\theta-1} \right) - \xi_p \left(\widehat{z}_{t-1} - \widehat{s}_{t-1} - \widehat{\lambda}_{p,t-1} \ln \widetilde{p}_*^{\theta-1} \right) \right]
\end{aligned}
$$

Let $A_* = \widetilde{p}_* - \frac{\theta}{\theta-1} \omega'_*(v_*^*) v_*^* mc_* (1-(1-\delta_z)\overline{\beta}\gamma)\theta$. Plugging this equation back into the pricing equation yields the NKPC:

$$
\begin{aligned}
& \frac{1}{1-\xi_p\overline{\beta}\gamma}\frac{\xi_p}{1-\xi_p}\left[\widehat{\pi}_t-\iota_p\widehat{\pi}_{t-1}+\frac{1}{\xi_p}\widehat{\lambda}_{p,t}\ln\widetilde{p}_*^{\theta-1}\right]A_* \\
& -\frac{1}{1-\xi_p\overline{\beta}\gamma}\frac{1}{1-\xi_p}B_*\left[\left(\widehat{z}_t-\widehat{s}_t-\widehat{\lambda}_{p,t}\ln\widetilde{p}_*^{\theta-1}\right)-\xi_p\left(\widehat{z}_{t-1}-\widehat{s}_{t-1}-\widehat{\lambda}_{p,t-1}\ln\widetilde{p}_*^{\theta-1}\right)\right]A_* \\
& -\frac{\theta}{\theta-1}(1-\delta_z)\overline{\beta}\gamma mc_* E_t\left[\widehat{\xi}_{t+1}-\widehat{\xi}_t+\widehat{\lambda}_{t+1}\right] \\
& -\frac{\theta}{\theta-1}\omega_*\left\{mc_*\widehat{\lambda}_t-(1-\delta_z)\overline{\beta}\gamma mc_* E_t\left[\widehat{\xi}_{t+1}-\widehat{\xi}_t+\widehat{\lambda}_{t+1}\right]\right\} \\
& -\frac{\theta}{\theta-1}\left\{\omega_* mc_*+(1-\omega_*)(1-\delta_z)\overline{\beta}\gamma mc_*\right\}\frac{1}{\theta}\widehat{\lambda}_{p,t} \\
& -\frac{\theta}{\theta-1}\omega_*'(v_*^*)v_*^* mc_*(1-(1-\delta_z)\overline{\beta}\gamma)\left(\widehat{z}_t-\widehat{s}_t-\widehat{\lambda}_{p,t}\ln\widetilde{p}_*^{\theta-1}\right) \\
& -\frac{\xi_p\overline{\beta}\gamma}{1-\xi_p\overline{\beta}\gamma}A_* E_t\left\{\begin{array}{c}\frac{\xi_p}{1-\xi_p}\left[\widehat{\pi}_{t+1}-\iota_p\widehat{\pi}_t+\frac{1}{\xi_p}\widehat{\lambda}_{p,t+1}\ln\widetilde{p}_*^{\theta-1}\right] \\ -\frac{1}{1-\xi_p}B_*\left[\left(\widehat{z}_{t+1}-\widehat{s}_{t+1}-\widehat{\lambda}_{p,t+1}\ln\widetilde{p}_*^{\theta-1}\right)-\xi_p\left(\widehat{z}_t-\widehat{s}_t-\widehat{\lambda}_{p,t}\ln\widetilde{p}_*^{\theta-1}\right)\right]\end{array}\right\} \\
& +E_t\frac{\xi_p\overline{\beta}\gamma}{1-\xi_p\overline{\beta}\gamma}\left[\iota_p\widehat{\pi}_t-\widehat{\pi}_{t+1}\right] \\
= \ & 0
\end{aligned}
$$

or after simplification

$$
\begin{aligned}
& \widehat{\pi}_t - \iota_p \widehat{\pi}_{t-1} \\
= \; & \overline{\beta}\gamma \frac{1-\xi_p(1-A_*)}{A_*} E_t \left[\widehat{\pi}_{t+1} - \iota_p \widehat{\pi}_t \right] \\
& + \frac{\left(1-\xi_p \overline{\beta}\gamma\right)\left(1-\xi_p\right)}{\xi_p A_*} \left[\frac{\theta}{\theta-1} \omega_* mc_* \widehat{\lambda}_t - \left(\omega_* - 1\right) \frac{\theta}{\theta-1} (1-\delta_z) \overline{\beta}\gamma mc_* E_t \left[\widehat{\xi}_{t+1} - \widehat{\xi}_t + \widehat{\lambda}_{t+1} \right] \right] \\
& - B_* \left(\widehat{z}_{t-1} - \widehat{s}_{t-1} - \widehat{\lambda}_{p,t-1} \ln \widetilde{p}_*^{\theta-1} \right) \\
& + \left\{ \frac{\left(1-\xi_p \overline{\beta}\gamma\right)\left(1-\xi_p\right)}{\xi_p \theta} \; \frac{\widetilde{p}_* - A_*}{A_*} + \frac{B_*}{\xi_p} + \xi_p \overline{\beta}\gamma B_* \right\} \left(\widehat{z}_t - \widehat{s}_t - \widehat{\lambda}_{p,t} \ln \widetilde{p}_*^{\theta-1} \right) \\
& - \overline{\beta}\gamma E_t \left(B_* \left(\widehat{z}_{t+1} - \widehat{s}_{t+1} \right) - \left(B_* + 1\right) \widehat{\lambda}_{p,t+1} \ln \widetilde{p}_*^{\theta-1} \right) \\
& + \left[-\frac{1}{\xi_p} \ln \widetilde{p}_*^{\theta-1} + \frac{\theta}{\theta-1} \left\{ \omega_* mc_* + (1-\omega_*)(1-\delta_z) \overline{\beta}\gamma mc_* \right\} \frac{1}{\theta} \frac{\left(1-\xi_p \overline{\beta}\gamma\right)\left(1-\xi_p\right)}{\xi_p A_*} \right] \widehat{\lambda}_{p,t}
\end{aligned}
$$

C. Summary

Modify resource constraint:

$$
inv_* = \left(1 - \frac{1-\delta_z}{\gamma}\right)(z_* - s_*)
$$

$$
c_* + i_* + y_* g_* = s_*
$$

$$
s_* + inv_* = y_*
$$

Add steady state equations for $\frac{z_*}{s_*}$, mc_* and $\widetilde{p}_*$:

$$
1 - F\left(\widetilde{p}_*^{\theta} \frac{z_*}{s_*}\right) = \frac{\left(1-(1-\delta_z)\overline{\beta}\gamma\right) mc_*}{\widetilde{p}_* - (1-\delta_z)\overline{\beta}\gamma mc_*} \tag{2}
$$

$$\widetilde{p}_*^{1-\theta}\Phi\left(\widetilde{p}_*^{\theta}\frac{z_*}{s_*}\right) = 1 \tag{3}$$

$$\begin{aligned}
& \widetilde{p}_* - \frac{\theta}{\theta-1}\left(\frac{v_*^*}{\Psi\left(p_*^{\theta}\frac{z_*}{s_*}\right)}\lambda_* + \left(1 - \frac{v_*^*}{\Psi\left(p_*^{\theta}\frac{z_*}{s_*}\right)}\right)(1-\delta_z)\overline{\beta}\gamma\lambda_*\right) \\
= & \ \widetilde{p}_* - \frac{\theta}{\theta-1}(1-\delta_z)\overline{\beta}\gamma\lambda_* - \frac{\theta}{\theta-1}\frac{v_*^*}{\Psi\left(p_*^{\theta}\frac{z_*}{s_*}\right)}\lambda_*\left(1-(1-\delta_z)\overline{\beta}\gamma\right) = 0 \\
\lambda_* = & \ \frac{\theta-1}{\theta}\frac{\widetilde{p}_*}{\left(\frac{v_*^*}{\Psi\left(p_*^{\theta}\frac{z_*}{s_*}\right)} + \left(1 - \frac{v_*^*}{\Psi\left(p_*^{\theta}\frac{z_*}{s_*}\right)}\right)(1-\delta_z)\overline{\beta}\gamma\right)} \qquad (4)
\end{aligned}$$

Equations (**??**)-(**??**) are solved jointly for $\frac{z_*}{s_*}$, λ_* and $\widetilde{p}_*$.

Denote

$$\begin{aligned}
v_*^* &= \widetilde{p}_*^{\theta}\frac{z_*}{s_*} \\
\Phi'(v^*) &= \frac{\theta-1}{\theta}(v^*)^{\frac{-1}{\theta}}\exp\left(\frac{\sigma^2}{2\theta^2}\right)\left(1-F\left(v^*-\sigma^2/\theta\right)\right) \\
\Psi(v^*) &= \int \min\left(v, v_t^*(i)\right) dF(v) \\
&= \exp\left(\frac{\sigma^2}{2}\right)F\left(v_t^*(i)-\sigma^2\right) + v_t^*(i)\left(1-F\left(v_t^*(i)\right)\right) \\
\Psi'(v^*) &= 1 - F\left(v_t^*(i)\right) \\
\omega(v^*) &= \frac{v^*}{\Psi(v^*)} \\
\omega'(v^*) &= \frac{\Psi(v^*) - v^*\Psi'(v^*)}{\left[\Psi(v^*)\right]^2}
\end{aligned}$$

$$
\begin{aligned}
A_* &= \widetilde{p}_* - \frac{\theta}{\theta-1}\omega_*'(v_*^*)v_*^* mc_*(1-(1-\delta_z)\overline{\beta}\gamma)\theta \\
B_* &= \frac{\widetilde{p}_*\Phi'(v_*^*)v_*^*}{(1-\theta)\,\Phi(v_*^*) + \theta\widetilde{p}_*\Phi'(v_*^*)v_*^*} \\
C_* &= \frac{f(v_*^*)v_*^*}{1-F(v_*^*)}\theta - \frac{\widetilde{p}_*}{\widetilde{p}_* - (1-\delta_z)\overline{\beta}\gamma mc_*}
\end{aligned}
$$

Add log-linearized resource constraint for $\widehat{inv}_t$ and $\widehat{z}_t$:

$$
inv_*\widehat{inv}_t = z_*\widehat{z}_t - s_*\widehat{s}_t - \frac{1-\delta_z}{\gamma}\left(z_*\widehat{z}_{t-1} - s_*\widehat{s}_{t-1}\right)
$$

$$
\frac{c_*}{y_*}\widehat{c}_t + \frac{i_*}{y_*}\widehat{i}_t + \widehat{g}_t + \frac{r_*^k k_*}{y_*}\widehat{u}_t = \frac{s_*}{y_*}\widehat{s}_t
$$

$$
\frac{s_*}{y_*}\widehat{s}_t + \frac{inv_*}{y_*}\widehat{inv}_t = \widehat{y}_t
$$

Log-linearized equations for I-S ratio

$$
\widehat{\lambda}_{t+j}\left(i\right) = \widehat{mc}_{t+j} + \eta\widehat{y}_{t+j}\left(i\right)
$$

$$
\begin{aligned}
&\left(C_*B_* - \frac{f(v_*^*)v_*^*}{1-F(v_*^*)}\right)\left(\widehat{z}_t - \widehat{s}_t - \widehat{\lambda}_{p,t}\ln\widetilde{p}_*^{\theta-1}\right) = C_*\widehat{\lambda}_{p,t}\ln\widetilde{p}_* \\
&+\frac{\widehat{\lambda}_t - (1-\delta_z)\overline{\beta}\gamma E_t\left(\widehat{\xi}_{t+1} - \widehat{\xi}_t + \widehat{\lambda}_{t+1}\right)}{1-(1-\delta_z)\overline{\beta}\gamma} + \frac{(1-\delta_z)\overline{\beta}\gamma mc_* E_t\left(\widehat{\xi}_{t+1} - \widehat{\xi}_t + \widehat{\lambda}_{t+1}\right)}{\widetilde{p}_* - (1-\delta_z)\overline{\beta}\gamma mc_*}
\end{aligned}
$$

and NKPC:

$$\begin{aligned}
& \widehat{\pi}_t - \iota_p \widehat{\pi}_{t-1} \\
= \; & \overline{\beta}\gamma \frac{1-\xi_p(1-A_*)}{A_*} E_t \left[\widehat{\pi}_{t+1} - \iota_p \widehat{\pi}_t \right] \\
& + \frac{\left(1-\xi_p \overline{\beta}\gamma\right)\left(1-\xi_p\right)}{\xi_p A_*} \left[\frac{\theta}{\theta-1} \omega_* mc_* \widehat{\lambda}_t - (\omega_* - 1) \frac{\theta}{\theta-1} (1-\delta_z) \overline{\beta}\gamma mc_* E_t \left[\widehat{\xi}_{t+1} - \widehat{\xi}_t + \widehat{\lambda}_{t+1} \right] \right] \\
& - B_* \left(\widehat{z}_{t-1} - \widehat{s}_{t-1} - \widehat{\lambda}_{p,t-1} \ln \widetilde{p}_*^{\theta-1} \right) \\
& + \left\{ \frac{\left(1-\xi_p \overline{\beta}\gamma\right)\left(1-\xi_p\right)}{\xi_p \theta} \; \frac{\widetilde{p}_* - A_*}{A_*} + \frac{B_*}{\xi_p} + \xi_p \overline{\beta}\gamma B_* \right\} \left(\widehat{z}_t - \widehat{s}_t - \widehat{\lambda}_{p,t} \ln \widetilde{p}_*^{\theta-1} \right) \\
& - \overline{\beta}\gamma E_t \left(B_* \left(\widehat{z}_{t+1} - \widehat{s}_{t+1} \right) - (B_* + 1) \widehat{\lambda}_{p,t+1} \ln \widetilde{p}_*^{\theta-1} \right) \\
& + \left[-\frac{1}{\xi_p} \ln \widetilde{p}_*^{\theta-1} + \frac{\theta}{\theta-1} \left\{ \omega_* mc_* + (1-\omega_*)(1-\delta_z) \overline{\beta}\gamma mc_* \right\} \frac{1}{\theta} \frac{\left(1-\xi_p \overline{\beta}\gamma\right)\left(1-\xi_p\right)}{\xi_p A_*} \right] \widehat{\lambda}_{p,t}
\end{aligned}$$