

# Appendix 1: Simple Calvo Economy

## Equilibrium system of equations

$$c_t = s_t$$

$$y_t = s_t + inv_t$$

$$inv_t = z_t - s_t - (1 - \delta)(z_{t-1} - s_{t-1})$$

$$W_t = W_{t-1}^\lambda (P_t c_t)^{1-\lambda}$$

$$M_t = P_t c_t$$

$$\mu_t = M_t / M_{t-1}$$

$$v_{t+s}^*(i) = \frac{z_{t+s}(i)}{s_{t+s}} \left( \frac{P_t(i)}{P_{t+s}} \right)^\theta$$

$$MC_t = W_t$$

$$\Lambda_t = MC_t$$

$$1 - F(v_{t+s}^*(i)) = \frac{\Lambda_{t+s} - (1 - \delta)E_{t+s} \frac{\beta U_{t+s+1} P_{t+s}}{U_{t+s} P_{t+s+1}} \Lambda_{t+s+1}}{P_t(i) - (1 - \delta)E_{t+s} \frac{\beta U_{t+s+1} P_{t+s}}{U_{t+s} P_{t+s+1}} \Lambda_{t+s+1}}$$

$$E_t \sum_{s=0}^{\infty} \xi_p^s \frac{\beta^s U_{t+s} P_t}{U_t P_{t+s}} \frac{Z_{t+s}(i)}{\omega_{t+s}(i)} \left\{ (1 - \theta) P_t(i) + \theta \Lambda_{t+s} \right. \\ \left. + \theta \omega_{t+s}(i) \left[ \Lambda_{t+s} - (1 - \delta) E_{t+s} \frac{\beta U_{t+s+1} P_{t+s}}{U_{t+s} P_{t+s+1}} \Lambda_{t+s+1} \right] \right\} = 0$$

$$P_t^{1-\theta} = \int_0^1 P_t^{1-\theta}(i) \Phi(v_t^*(i)) di$$

## Normalization

$$w_t = W_t/P_t, \quad m_t = M_t/P_t, \quad \text{etc.}$$

## Stationary equations

$$inv_t = z_t - s_t - (1 - \delta)(z_{t-1} - s_{t-1})$$

$$c_t = s_t$$

$$s_t + inv_t = y_t$$

$$w_t = w_{t-1}^\lambda \pi_t^{-\lambda} c_t^{1-\lambda}$$

$$m_t = c_t$$

$$\mu_t = m_t/m_{t-1}\pi_t$$

$$mc_t = w_t$$

$$\lambda_t = mc_t$$

$$v_{t+s}^*(i) = p_t^\theta(i) \frac{z_{t+s}(i)}{s_{t+s}}$$

$$1 - F(v_{t+s}^*(i)) = \frac{\lambda_{t+s} - (1 - \delta)E_{t+s} \left( \frac{\beta c_{t+s}}{c_{t+s+1}} \lambda_{t+s+1} \right)}{p_t(i) - (1 - \delta)E_{t+s} \left( \frac{\beta c_{t+s}}{c_{t+s+1}} \lambda_{t+s+1} \right)}$$

$$\begin{aligned}
& \frac{z_t(i)}{\omega_t(i)} (\theta - 1) \left[ p_t(i) - \frac{\theta}{\theta - 1} \left\{ \omega_t(i) \lambda_t + (1 - \omega_t(i))(1 - \delta) E_t \frac{\beta \xi_{t+1}}{\xi_t} \lambda_{t+1} \right\} \right] \\
& + E_t \sum_{s=1}^{\infty} \xi_p^s \frac{\beta^s c_t z_{t+s}(i)}{c_{t+s} \omega_{t+s}(i)} (\theta - 1) \times \\
& \times \left[ p_t(i) - \frac{\theta}{\theta - 1} \left\{ \omega_{t+s}(i) \lambda_{t+s} + (1 - \omega_{t+s}(i))(1 - \delta) E_{t+s} \frac{\beta c_{t+s}}{c_{t+s+1}} \lambda_{t+s+1} \right\} \right] \\
& = 0
\end{aligned}$$

where  $\omega_{t+s}(i) = \frac{v_{t+s}^*(i)}{\int \min(v, v_{t+s}^*(i)) dF(v)}$ .

$$\begin{aligned}
1 &= \int_0^1 p_t^{1-\theta}(i) \Phi(v_t^*(i)) di \\
&= (1 - \xi_p) p_t^{1-\theta}(i) \Phi(v_t^*(i)) + (1 - \xi_p) \sum_{s=1}^{\infty} \xi_p^s p_{t-s}^{1-\theta}(i) \Phi(v_t^*(i))
\end{aligned}$$

## Steady state

$$inv_* = \delta (z_* - s_*)$$

$$c_* = s_*$$

$$s_* + inv_* = y_*$$

$$w_* = \pi_*^{-\lambda} c_*$$

$$m_* = c_*$$

$$\mu = \pi_*$$

$$mc_* = w_*$$

$$\lambda_* = mc_*$$

$$v_*^* = p_*^\theta \frac{z_*^*}{s_*}$$

$$1 - F(v_*^*) = \frac{(1 - (1 - \delta)\beta) mc_*}{p_* - (1 - \delta)\beta mc_*}$$

$$p_* = \frac{\frac{\theta}{\theta-1}(1 - \delta)\beta \gamma mc_*}{1 - \frac{\theta}{\theta-1} \frac{z_*^*}{s_*} mc_* (1 - (1 - \delta)\beta)}$$

$$p_*^{1-\theta} \Phi\left(p_*^\theta \frac{z_*^*}{s_*}\right) = 1$$

Recall

$$\begin{aligned} \Phi(v^*) &= \int_{-\infty}^{\infty} \min(v, v^*) dF(v) \\ &= \exp\left(\frac{\sigma^2}{2}\right) F(v^* - \sigma^2) + v^* \frac{\theta-1}{\theta} \exp\left(\frac{\sigma^2}{2\theta^2}\right) (1 - F(v^* - \sigma^2/\theta)) \end{aligned}$$

$$\Phi'(v^*) = \frac{\theta-1}{\theta} (v^*)^{\frac{-1}{\theta}} \exp\left(\frac{\sigma^2}{2\theta^2}\right) (1 - F(v^* - \sigma^2/\theta))$$

$$\omega(v^*) = \frac{v^*}{\Phi(v^*)}$$

$$\omega'(v^*) = \frac{\Phi(v^*) - v^* \Phi'(v^*)}{[\Phi(v^*)]^2}$$

$$A_* = p_* - \frac{\theta}{\theta-1} \omega'(v_*^*) v_*^* mc_* (1 - (1 - \delta)\beta) \theta$$

$$B_* = \frac{p_* \Phi'(v_*^*) v_*^*}{(1 - \theta) \Phi(v_*^*) + \theta p_* \Phi'(v_*^*) v_*^*}$$

$$C_* = \frac{f(v_*^*) v_*^*}{1 - F(v_*^*)} \theta - \frac{p_*}{p_* - (1 - \delta)\beta mc_*}$$

**Log-linearized equations (including aggregation)**

$$inv_* \widehat{inv}_t = z_* \widehat{z}_t - s_* \widehat{s}_t - (1 - \delta) (z_* \widehat{z}_{t-1} - s_* \widehat{s}_{t-1})$$

$$\widehat{c}_t = \widehat{s}_t$$

$$\frac{s_* \widehat{s}_t}{y_*} + \frac{inv_* \widehat{inv}_t}{y_*} = \widehat{y}_t$$

$$\widehat{w}_t = \lambda \widehat{w}_{t-1} - \lambda \widehat{\pi}_t + (1 - \lambda) \widehat{c}_t$$

$$\widehat{m}_t = \widehat{c}_t$$

$$\widehat{\mu}_t = \widehat{m}_t - \widehat{m}_{t-1} + \widehat{\pi}_t$$

$$\widehat{\mu}_t = \rho_\mu \widehat{\mu}_{t-1} + \varepsilon_t^\mu$$

$$\widehat{m}c_t = \widehat{w}_t$$

$$\widehat{\lambda}_t = \widehat{m}c_t$$

First derive the equation for I-S ratios:

$$\begin{aligned} \widehat{v}_{t+s}^*(i) &= \theta \left( \widehat{p}_t(i) - \sum_{l=1}^s \widehat{\pi}_{t+l} \right) + \widehat{z}_{t+s}(i) - \widehat{s}_{t+s} \\ &= \theta \left( \widehat{\widehat{p}}_t(i) - \sum_{l=1}^s \widehat{\pi}_{t+l} \right) + \widehat{z}_{t+s}(i) - \widehat{s}_{t+s} \end{aligned}$$

Recall I-S equation

$$1 - F(v_{t+s}^*(i)) = \frac{\lambda_{t+s} - (1 - \delta) E_{t+s} \left( \frac{\beta c_{t+s}}{c_{t+s+1}} \lambda_{t+s+1} \right)}{p_t(i) \prod_{l=1}^s \pi_{t+l} - (1 - \delta) E_{t+s} \left( \frac{\beta c_{t+s}}{c_{t+s+1}} \lambda_{t+s+1} \right)}$$

log-linearizing gives:

$$\begin{aligned}
& \frac{f(v_*^*)v_*^*}{1-F(v_*^*)} \left( \widehat{s}_{t+s} - \widehat{z}_{t+s}(i) - \theta \left( \widehat{p}_t(i) - \sum_{l=1}^s \widehat{\pi}_{t+l} \right) \right) \\
= & \frac{\widehat{\lambda}_{t+s} - (1-\delta)\beta E_{t+s} \left( -\widehat{c}_{t+s+1} + \widehat{c}_{t+s} + \widehat{\lambda}_{t+s+1} \right)}{1 - (1-\delta)\beta} \\
& - \frac{p_* \left( \widehat{p}_t(i) - \sum_{l=1}^s \widehat{\pi}_{t+l} \right) - (1-\delta)\beta m c_* E_{t+s} \left( -\widehat{c}_{t+s+1} + \widehat{c}_{t+s} + \widehat{\lambda}_{t+s+1} \right)}{p_* - (1-\delta)\beta m c_*}
\end{aligned}$$

Sum across  $i$  and use aggregate price equation

$$\begin{aligned}
& \left\{ \widehat{p}_t(i) + \sum_{s=1}^{\infty} \xi_p^s \left( \widehat{p}_{t-s}(i) - \sum_{l=1}^s \widehat{\pi}_{t-l+1} \right) \right\} (1 - \xi_p) \\
= & - \frac{p_* \Phi'(v_*^*)v_*^*}{(1-\theta)\Phi(v_*^*) + \theta p_* \Phi'(v_*^*)v_*^*} (\widehat{z}_t - \widehat{s}_t)
\end{aligned}$$

to obtain

$$\begin{aligned}
& \frac{f(v_*^*)v_*^*}{1-F(v_*^*)} \left( \frac{\theta p_* \Phi'(v_*^*)v_*^*}{(1-\theta)\Phi(v_*^*) + \theta p_* \Phi'(v_*^*)v_*^*} - 1 \right) (\widehat{z}_t - \widehat{s}_t) \\
= & \frac{\widehat{\lambda}_t - (1-\delta)\beta E_t \left( -\widehat{c}_{t+1} + \widehat{c}_t + \widehat{\lambda}_{t+1} \right)}{1 - (1-\delta)\beta} \\
& + \frac{p_*}{p_* - (1-\delta)\beta m c_*} \frac{p_* \Phi'(v_*^*)v_*^*}{(1-\theta)\Phi(v_*^*) + \theta p_* \Phi'(v_*^*)v_*^*} (\widehat{z}_t - \widehat{s}_t) \\
& + \frac{(1-\delta)\beta m c_* E_t \left( -\widehat{c}_{t+1} + \widehat{c}_t + \widehat{\lambda}_{t+1} \right)}{p_* - (1-\delta)\beta m c_*}
\end{aligned}$$

Let  $B_* = \frac{p_* \Phi'(v_*^*)v_*^*}{(1-\theta)\Phi(v_*^*) + \theta p_* \Phi'(v_*^*)v_*^*}$ ,  $C_* = \frac{f(v_*^*)v_*^*}{1-F(v_*^*)} \theta - \frac{p_*}{p_* - (1-\delta)\beta m c_*}$ . Simplify by gathering  $\widehat{z}_t - \widehat{s}_t$

term:

$$\begin{aligned} \left( C_* B_* - \frac{f(v_*) v_*^*}{1 - F(v_*)} \right) (\widehat{z}_t - \widehat{s}_t) &= \frac{\widehat{\lambda}_t - (1 - \delta) \beta E_t \left( -\widehat{c}_{t+1} + \widehat{c}_t + \widehat{\lambda}_{t+1} \right)}{1 - (1 - \delta) \beta} \\ &+ \frac{(1 - \delta) \beta m c_* E_t \left( -\widehat{c}_{t+1} + \widehat{c}_t + \widehat{\lambda}_{t+1} \right)}{p_* - (1 - \delta) \beta m c_*} \end{aligned}$$

or after separating markup term from mc-growth term:

$$\begin{aligned} \left( C_* B_* - \frac{f(v_*) v_*^*}{1 - F(v_*)} \right) (\widehat{z}_t - \widehat{s}_t) &= \frac{m c_*}{p_* - (1 - \delta) \beta m c_*} \widehat{\lambda}_t \\ &+ \frac{p_*/m c_* - 1}{(1 - (1 - \delta) \beta) (p_* - (1 - \delta) \beta m c_*)} \left[ \widehat{\lambda}_t - (1 - \delta) \beta E_t \left( -\widehat{c}_{t+1} + \widehat{c}_t + \widehat{\lambda}_{t+1} \right) \right] \end{aligned}$$

Turning to the pricing equation:

$$\begin{aligned} &\frac{z_t(i)}{\omega_t(i)} \left[ p_t(i) - \frac{\theta}{\theta - 1} \left\{ \omega_t(i) \lambda_t + (1 - \omega_t(i)) (1 - \delta) E_t \frac{\beta c_t}{c_{t+1}} \lambda_{t+1} \right\} \right] \\ &+ E_t \sum_{s=0}^{\infty} \xi_p^s \frac{\beta^s c_t}{c_{t+s}} \frac{z_{t+s}(i)}{\omega_{t+s}(i)} \\ &\left[ p_t(i) \frac{1}{\prod_{l=1}^s \pi_{t+l}} + \frac{\theta}{\theta - 1} \left\{ \omega_{t+s}(i) \lambda_{t+s} + (1 - \omega_{t+s}(i)) (1 - \delta) E_{t+s} \frac{\beta c_t}{c_{t+1}} \lambda_{t+s+1} \right\} \right] \\ &= 0 \end{aligned}$$

where  $\omega_{t+s}(i) = \frac{v_{t+s}^*(i)}{\int \min(v, v_{t+s}^*(i)) dF(v)}$ . Next log-linearize the pricing equation:

$$\begin{aligned}
& p_* \widehat{p}_t(i) - \frac{\theta}{\theta - 1} (1 - \delta) \beta m c_* E_t \left[ -\widehat{c}_{t+s+1} + \widehat{c}_{t+s} + \widehat{\lambda}_{t+s+1} \right] \\
& - \frac{\theta}{\theta - 1} \omega_* \left\{ m c_* \widehat{m} c_t - (1 - \delta) \beta m c_* E_t \left[ -\widehat{c}_{t+s+1} + \widehat{c}_{t+s} + \widehat{\lambda}_{t+s+1} \right] \right\} \\
& - \frac{\theta}{\theta - 1} \omega'_*(v_*^*) v_*^* m c_* (1 - (1 - \delta) \beta) (\theta \widehat{p}_t(i) + \widehat{z}_t(i) - \widehat{s}_t) \\
& + E_t \sum_{s=1}^{\infty} \xi_p^s \beta^s \left\{ p_* \widehat{p}_t(i) - \sum_{l=1}^s \widehat{\pi}_{t+l} - \frac{\theta}{\theta - 1} (1 - \delta) \beta m c_* \left[ -\widehat{c}_{t+s+1} + \widehat{c}_{t+s} + \widehat{\lambda}_{t+s+1} \right] \right. \\
& - \frac{\theta}{\theta - 1} \omega_* \left\{ m c_* \widehat{\lambda}_{t+s} - (1 - \delta) \beta m c_* E_t \left[ -\widehat{c}_{t+s+1} + \widehat{c}_{t+s} + \widehat{\lambda}_{t+s+1} \right] \right\} \\
& \left. - \frac{\theta}{\theta - 1} \omega'_*(v_*^*) v_*^* m c_* (1 - (1 - \delta) \beta) \left( \theta \left( \widehat{p}_t(i) - \sum_{l=1}^s \widehat{\pi}_{t+l} \right) + \widehat{z}_{t+s}(i) - \widehat{s}_{t+s} \right) \right\} \\
& = 0
\end{aligned}$$

where  $\omega_* = \frac{v_*^*}{\Phi(v_*^*)}$ ,  $\omega'_* = \frac{\Phi(v_*^*) - v_*^* \Phi'(v_*^*)}{[\Phi(v_*^*)]^2}$ ,

minus same at  $t + 1$  multiplied by  $\xi_p \beta$  gives

$$\begin{aligned}
& \widehat{p}_t(i) \left( p_* - \frac{\theta}{\theta - 1} \omega'_*(v_*^*) v_*^* m c_* (1 - (1 - \delta) \beta) \theta \right) - \frac{\theta}{\theta - 1} (1 - \delta) \beta m c_* E_t \left[ -\widehat{c}_{t+1} + \widehat{c}_t + \widehat{\lambda}_{t+1} \right] \\
& - \frac{\theta}{\theta - 1} \omega_* \left\{ m c_* \widehat{\lambda}_t - (1 - \delta) \beta m c_* E_t \left[ -\widehat{c}_{t+1} + \widehat{c}_t + \widehat{\lambda}_{t+1} \right] \right\} \\
& - \frac{\theta}{\theta - 1} \omega'_*(v_*^*) v_*^* m c_* (1 - (1 - \delta) \beta) (\widehat{z}_t - \widehat{s}_t) \\
& - \frac{\xi_p \beta}{1 - \xi_p \beta} \left( p_* - \frac{\theta}{\theta - 1} \omega'_*(v_*^*) v_*^* m c_* (1 - (1 - \delta) \beta) \theta \right) E_t \widehat{p}_{t+1}(i) \\
& + E_t \frac{\xi_p \beta}{1 - \xi_p \beta} \left( \left( p_* - \frac{\theta}{\theta - 1} \omega'_*(v_*^*) v_*^* m c_* (1 - (1 - \delta) \beta) \theta \right) \widehat{p}_t(i) - \widehat{\pi}_{t+1} \right) \\
& = 0
\end{aligned}$$

The aggregate price is determined by

$$\begin{aligned}
1 &= \int_0^1 p_t^{1-\theta}(i) \Phi(v_t^*(i)) di \\
&= (1 - \xi_p) p_t^{1-\theta}(i) \Phi(v_t^*(i)) + (1 - \xi_p) \sum_{s=1}^{\infty} \xi_p^s p_{t-s}^{1-\theta}(i) \left( \frac{1}{\prod_{l=1}^s \pi_{t-l+1}} \right)^{1-\theta} \Phi(v_t^*(i))
\end{aligned}$$

which after log-linearization becomes

$$\begin{aligned}
0 &= (1 - \theta) \hat{p}_t(i) \Phi(v_*^*) + p_*^{1-\theta} \Phi'(v_*^*) v_*^* (\theta \hat{p}_t(i) + \hat{z}_t(i) - \hat{s}_t) \\
&\quad + (1 - \theta) \sum_{s=1}^{\infty} \xi_p^s \left( \hat{p}_{t-s}(i) - \sum_{l=1}^s \hat{\pi}_{t-l+1} \right) \Phi(v_*^*) \\
&\quad + \sum_{s=1}^{\infty} \xi_p^s \left[ p_*^{1-\theta} \Phi'(v_*^*) v_*^* \left( \theta \hat{p}_{t-s}(i) - \theta \sum_{l=1}^s \hat{\pi}_{t-l+1} + \hat{z}_t(i) - \hat{s}_t \right) \right]
\end{aligned}$$

$$\begin{aligned}
0 &= \hat{p}_t(i) + \frac{1}{1 - \xi_p} \frac{p_* \Phi'(v_*^*) v_*^*}{(1 - \theta) \Phi(v_*^*) + \theta p_* \Phi'(v_*^*) v_*^*} (\hat{z}_t - \hat{s}_t) \\
&\quad + \sum_{s=1}^{\infty} \xi_p^s \left( \hat{p}_{t-s}(i) - \sum_{l=1}^s \hat{\pi}_{t-l+1} \right)
\end{aligned}$$

minus same at  $t - 1$  multiplied by  $\xi_p$  :

$$\hat{p}_t(i) = \frac{\xi_p}{1 - \xi_p} \hat{\pi}_t - \frac{1}{1 - \xi_p} \frac{p_* \Phi'(v_*^*) v_*^*}{(1 - \theta) \Phi(v_*^*) + \theta p_* \Phi'(v_*^*) v_*^*} [(\hat{z}_t - \hat{s}_t) - \xi_p (\hat{z}_{t-1} - \hat{s}_{t-1})]$$

Let  $A_* = p_* - \frac{\theta}{\theta-1} \omega'_*(v_*^*) v_*^* m c_*(1 - (1 - \delta)\beta)\theta$ . Plugging this equation back into the pricing equation yields the NKPC:

$$\begin{aligned}
& \frac{1}{1 - \xi_p \beta} \frac{\xi_p}{1 - \xi_p} \widehat{\pi}_t A_* \\
& - \frac{1}{1 - \xi_p \beta} \frac{1}{1 - \xi_p} B_* [(\widehat{z}_t - \widehat{s}_t) - \xi_p (\widehat{z}_{t-1} - \widehat{s}_{t-1})] A_* \\
& - \frac{\theta}{\theta - 1} (1 - \delta) \beta m c_* E_t [-\widehat{c}_{t+1} + \widehat{c}_t + \widehat{\lambda}_{t+1}] \\
& - \frac{\theta}{\theta - 1} \omega_* \left\{ m c_* \widehat{\lambda}_t - (1 - \delta) \beta m c_* E_t [-\widehat{c}_{t+1} + \widehat{c}_t + \widehat{\lambda}_{t+1}] \right\} \\
& - \frac{\theta}{\theta - 1} \omega'_*(v_*^*) v_*^* m c_* (1 - (1 - \delta) \beta) (\widehat{z}_t - \widehat{s}_t) \\
& - \frac{\xi_p \beta}{1 - \xi_p \beta} A_* E_t \left\{ \frac{\xi_p}{1 - \xi_p} \widehat{\pi}_{t+1} - \frac{1}{1 - \xi_p} B_* [(\widehat{z}_{t+1} - \widehat{s}_{t+1}) - \xi_p (\widehat{z}_t - \widehat{s}_t)] \right\} \\
& - E_t \frac{\xi_p \beta}{1 - \xi_p \beta} \widehat{\pi}_{t+1} \\
& = 0
\end{aligned}$$

or after simplification

$$\begin{aligned}
\widehat{\pi}_t &= \beta \frac{1 - \xi_p (1 - A_*)}{A_*} E_t \widehat{\pi}_{t+1} \\
&+ \frac{(1 - \xi_p \beta) (1 - \xi_p)}{\xi_p A_*} \left[ \frac{\theta}{\theta - 1} \omega_* m c_* \widehat{\lambda}_t - (\omega_* - 1) \frac{\theta}{\theta - 1} (1 - \delta) \beta m c_* E_t [-\widehat{c}_{t+1} + \widehat{c}_t + \widehat{\lambda}_{t+1}] \right] \\
&- B_* (\widehat{z}_{t-1} - \widehat{s}_{t-1}) \\
&+ \left\{ \frac{(1 - \xi_p \beta) (1 - \xi_p)}{\xi_p \theta} \frac{p_* - A_*}{A_*} + \frac{B_*}{\xi_p} + \xi_p \beta B_* \right\} (\widehat{z}_t - \widehat{s}_t) \\
&- \beta B_* E_t (\widehat{z}_{t+1} - \widehat{s}_{t+1})
\end{aligned}$$

### Adding adjustment cost

In retailer's problem we add convex adjustment cost in the law of motion for inventory

stock:

$$z_{t+s}(i) = (1 - \delta) \left( z_{t+s-1}(i) - s_{t+s-1} \left( \frac{P_t(i)}{P_{t+s-1}} \right)^{-\theta} \int \min(v, v_{t+s}^*(i)) dF(v) \right) + y_{t+s}(i) - \frac{\eta}{2} (y_{t+s}(i) - (I_\eta y_{t+s-1}(i) + (1 - I_\eta) y_*))^2 [I_\eta y_{t+s}(i) + (1 - I_\eta) y_*]^{-1} ,$$

where  $I_\eta = \{0, 1\}$ . The adjustment cost is a convex function of the output level deviations from steady state if  $I_\eta = 0$ , and if  $I_\eta = 1$  the cost a function of the *change* in the output level. The new version of equation for the log-linearized marginal cost of the new stock is

$$\widehat{\lambda}_{t+s}(i) = \widehat{m}c_{t+s} + \eta [\widehat{y}_{t+s}(i) - I_\eta \widehat{y}_{t+s-1}(i)] .$$

## Appendix 2: Adding inventories to Smets and Wouters (2007) model

### A. New equations

Two equations are added (inventory-sales ratio equation, law of motion for aggregate inventories), and two equations are modified (the pricing equation and the resource constraint).

#### Resource constraint:

$$\Pi_t = \int \Pi_t(i) di = \int P_t(i) S_t(i) di - W_t L_t - R_t^k K_t$$

taking into account that

$$\int P_t(i) S_t(i) di = P_t S_t = P_t Y_t - P_t Inv_t$$

where  $Inv_t$  is period t inventory investment (change in the end-of-period stock):

$$\begin{aligned} Inv_t &= \int \left[ Z_t(i) - S_t(i) - (1 - \delta_z)(Z_{t-1}(i) - S_{t-1}(i)) + \frac{\eta}{2}(Y_t(i) - Y_{t-1}(i))^2 Y_t^{-1}(i) \right] di \\ &= Z_t - S_t - (1 - \delta_z)(Z_{t-1} - S_{t-1}) + \frac{\eta}{2} \int (Y_t(i) - Y_{t-1}(i))^2 Y_t^{-1}(i) di \end{aligned}$$

Hence the resource constraint is

$$C_t + I_t + Inv_t + G_t + a(u_t) \bar{K}_{t-1} = Y_t$$

and final sales to domestic purchasers are defined as

$$C_t + I_t + G_t + a(u_t) \bar{K}_{t-1} = S_t$$

**Final good producers:**

$$\begin{aligned}
& \max_{S_t, S_t(i, v)} P_t S_t - \int_0^1 P_t(i) S_t(i, v) di \\
& s.t. \quad S_t = \left[ \int_0^1 \left( v_t(i)^{\frac{1}{\theta_t}} S_t(i, v)^{\frac{\theta_t-1}{\theta_t}} \right) di \right]^{\frac{\theta_t}{\theta_t-1}} \\
& \quad \quad S_t(i, v) \leq Z_t(i) \quad [\tilde{\gamma}_t(i, v)]
\end{aligned}$$

FOCs:

$$\begin{aligned}
P_t v_t(i)^{\frac{1}{\theta_t}} S_t(i, v)^{\frac{-1}{\theta_t}} S_t^{\frac{1}{\theta_t}} - P_t(i) - \tilde{\gamma}_t(i, v) &= 0 \\
\tilde{\gamma}_t(i, v) (Z_t(i) - S_t(i, v)) &= 0
\end{aligned} \tag{3}$$

Case 1:  $\tilde{\gamma}_t(i, v) > 0$ , then  $S_t(i, v) = Z_t(i)$ , and  $\tilde{\gamma}_t(i, v) = P_t v_t(i)^{\frac{1}{\theta_t}} Z_t(i)^{\frac{-1}{\theta_t}} S_t^{\frac{1}{\theta_t}} - P_t(i)$

Case 2:  $\tilde{\gamma}_t(i, v) = 0$ , then  $S_t(i, v) = v_t(i) S_t \left( \frac{P_t(i)}{P_t} \right)^{-\theta_t}$ .

**Intermediate good producers:**

$$\begin{aligned}
& \max_{\tilde{P}_t(i), \{Z_{t+s}(i)\}} E_t \sum_{s=0}^{\infty} \xi_p^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \times \\
& \left[ \tilde{P}_t(i) X_{t,s} S_{t+s} \left( \frac{\tilde{P}_t(i) X_{t,s}}{P_{t+s}} \right)^{-\theta_{t+s}} \int \min(v, v_{t+s}^*(i)) dF(v) - MC_{t+s} Y_{t+s}(i) \right] \\
& + E_t \sum_{s=1}^{\infty} \xi_p^{s-1} (1 - \xi_p) \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} [-MC_{t+s} Y_{t+s}(i)] \\
s.t. \quad & v_{t+s}^*(i) = \frac{Z_{t+s}(i)}{S_{t+s}} \left( \frac{\tilde{P}_t(i) X_{t,s}}{P_{t+s}} \right)^{\theta_{t+s}} \\
& Z_{t+s}(i) = (1 - \delta_z) \left( Z_{t+s-1}(i) - S_{t+s-1} \left( \frac{\tilde{P}_t(i) X_{t,s-1}}{P_{t+s-1}} \right)^{-\theta_{t+s-1}} \int \min(v, v_{t+s-1}^*(i)) dF(v) \right) + \\
& - \frac{\eta}{2} (Y_{t+s}(i) - \gamma Y_{t+s-1}(i))^2 Y_{t+s}^{-1}(i)
\end{aligned}$$

Using the fact that

$$\frac{d}{dz} \int \min(v, v^*(z)) dF(v) = (1 - F(v^*(z))) \frac{dv^*(z)}{dz}$$

$$[Y] : -MC_{t+s} + \lambda_{t+s}(i) \left( 1 - \eta \left( 1 - \gamma \frac{Y_{t+s-1}(i)}{Y_{t+s}(i)} \right) + \frac{\eta}{2} \left( 1 - \gamma \frac{Y_{t+s-1}(i)}{Y_{t+s}(i)} \right)^2 \right) = 0$$

So that the shadow value of stock  $\lambda_{t+s}(i) = MC_{t+s} / \left[ 1 - \eta \left( 1 - \gamma \frac{Y_{t+s-1}(i)}{Y_{t+s}(i)} \right) + \frac{\eta}{2} \left( 1 - \gamma \frac{Y_{t+s-1}(i)}{Y_{t+s}(i)} \right)^2 \right]$ .

FOCs for  $Z_{t+s}(i)$

$$1 - F(v_{t+s}^*(i)) = \frac{\lambda_{t+s}(i) - (1 - \delta_z) E_{t+s} \frac{\beta \Xi_{t+s+1} P_{t+s}}{\Xi_{t+s} P_{t+s+1}} \lambda_{t+s+1}(i)}{\tilde{P}_t(i) X_{t,s} - (1 - \delta_z) E_{t+s} \frac{\beta \Xi_{t+s+1} P_{t+s}}{\Xi_{t+s} P_{t+s+1}} \lambda_{t+s+1}(i)}$$

FOCs for  $\tilde{P}_t(i)$  :

$$\begin{aligned}
& E_t \sum_{s=0}^{\infty} \xi_p^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \times \\
& \left[ (1 - \theta_t) \tilde{P}_t(i) X_{t,s} S_{t+s} \left( \frac{\tilde{P}_t(i) X_{t,s}}{P_{t+s}} \right)^{-\theta_{t+s}} \int \min(v, v_{t+s}^*(i)) dF(v) + \theta_{t+s} \tilde{P}_t(i) X_{t,s} Z_{t+s}(i) (1 - F(v_{t+s}^*(i))) \right. \\
& \quad \left. + (1 - \delta_z) E_{t+s} \frac{\beta \Xi_{t+s+1} P_{t+s}}{\Xi_{t+s} P_{t+s+1}} \lambda_{t+s+1}(i) \times \right. \\
& \quad \left. \left\{ \theta_{t+s} S_{t+s} \left( \frac{\tilde{P}_t(i) X_{t,s}}{P_{t+s}} \right)^{-\theta_{t+s}} \int \min(v, v_{t+s}^*(i)) dF(v) - \theta_{t+s} Z_{t+s}(i) (1 - F(v_{t+s}^*(i))) \right\} \right] \\
& = E_t \sum_{s=0}^{\infty} \xi_p^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \frac{Z_{t+s}(i)}{v_{t+s}^*(i)} \times \\
& \left[ (1 - \theta_{t+s}) \tilde{P}_t(i) X_{t,s} \int \min(v, v_{t+s}^*(i)) dF(v) + \theta_{t+s} \tilde{P}_t(i) X_{t,s} v_{t+s}^*(i) (1 - F(v_{t+s}^*(i))) \right. \\
& \quad \left. + (1 - \delta_z) E_{t+s} \frac{\beta \Xi_{t+s+1} P_{t+s}}{\Xi_{t+s} P_{t+s+1}} \lambda_{t+s+1}(i) \times \right. \\
& \quad \left. \left\{ \theta_{t+s} \int \min(v, v_{t+s}^*(i)) dF(v) - \theta_{t+s} v_{t+s}^*(i) (1 - F(v_{t+s}^*(i))) \right\} \right] \\
& = E_t \sum_{s=0}^{\infty} \xi_p^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} Z_{t+s}(i) \frac{\int \min(v, v_{t+s}^*(i)) dF(v)}{v_{t+s}^*(i)} \times \\
& \left[ (1 - \theta_{t+s}) \tilde{P}_t(i) X_{t,s} + \theta_{t+s} (1 - \delta_z) E_{t+s} \frac{\beta \Xi_{t+s+1} P_{t+s}}{\Xi_{t+s} P_{t+s+1}} \lambda_{t+s+1}(i) \right. \\
& \quad \left. + \theta_{t+s} \left\{ \lambda_{t+s}(i) - (1 - \delta_z) E_{t+s} \frac{\beta \Xi_{t+s+1} P_{t+s}}{\Xi_{t+s} P_{t+s+1}} \lambda_{t+s+1}(i) \right\} \frac{v_{t+s}^*(i)}{\int \min(v, v_{t+s}^*(i)) dF(v)} \right]
\end{aligned}$$

For the aggregate price we obtain

$$\begin{aligned}
P_t^{1-\theta} & = \left[ \int_{v_t(i) \leq v_t^*(i)} v_t(i) P_t(i)^{1-\theta} di + \int_{v_t(i) > v_t^*(i)} v_t(i) \left( \frac{v_t^*(i)}{v_t(i)} \right)^{\frac{\theta-1}{\theta}} P_t(i)^{1-\theta} di \right] \\
& = \int_0^1 P_t^{1-\theta}(i) \Phi(v_t^*(i)) di
\end{aligned}$$

where we invoke the law of large numbers and the fact that  $v_t(i)$  is independent of  $P_t(i)$  and  $z_t(i)$ . Here  $\Phi(v_t^*(i))$  satisfies:

$$\Phi(v_t^*(i)) = \int_{v_t(i) \leq v_t^*(i)} v_t(i) dF(v) + v_t^*(i)^{\frac{\theta-1}{\theta}} \int_{v_t(i) > v_t^*(i)} v_t(i)^{\frac{1}{\theta}} dF(v)$$

For normally distributed demand shocks

$$\Phi(v_t^*(i)) = \exp\left(\frac{\sigma^2}{2}\right) F(v_t^*(i) - \sigma^2) + v_t^*(i)^{\frac{\theta-1}{\theta}} \exp\left(\frac{\sigma^2}{2\theta^2}\right) (1 - F(v_t^*(i) - \sigma^2/\theta))$$

which implies

$$\begin{aligned}\Phi'(v^*) &= \frac{\theta-1}{\theta} (v^*)^{\frac{-1}{\theta}} \exp\left(\frac{\sigma^2}{2\theta^2}\right) (1 - F(v^* - \sigma^2/\theta)) \\ \Psi(v^*) &= \int \min(v, v_t^*(i)) dF(v) \\ &= \exp\left(\frac{\sigma^2}{2}\right) F(v_t^*(i) - \sigma^2) + v_t^*(i) (1 - F(v_t^*(i))) \\ \Psi'(v^*) &= 1 - F(v_t^*(i)) \\ \omega(v^*) &= \frac{v^*}{\Psi(v^*)} \\ \omega'(v^*) &= \frac{\Psi(v^*) - v^* \Psi'(v^*)}{[\Psi(v^*)]^2}\end{aligned}$$

Stationary equations

$$inv_t = z_t - s_t - \frac{1 - \delta_z}{\gamma} (z_{t-1} - s_{t-1}) + \frac{\eta}{2} \int (y_t(i) - y_{t-1}(i))^2 y_t^{-1}(i) di$$

$$c_t + i_t + y_* g_t + \frac{a(u_t) \bar{k}_{t-1}}{\gamma} = s_t$$

$$s_t + inv_t = y_t$$

$$v_{t+s}^*(i) = p_t^{\theta_{t+s}}(i) \frac{z_{t+s}(i)}{s_{t+s}}$$

$$\lambda_{t+s}(i) = mc_{t+s}/ \left[ 1 - \eta \left( 1 - \frac{y_{t+s-1}(i)}{y_{t+s}(i)} \right) + \frac{\eta}{2} \left( 1 - \frac{y_{t+s-1}(i)}{y_{t+s}(i)} \right)^2 \right]$$

$$1 - F(v_{t+s}^*(i)) = \frac{\lambda_{t+s} - (1 - \delta_z) E_{t+s} \left( \frac{\bar{\beta} \gamma \xi_{t+s+1}}{\xi_{t+s}} \lambda_{t+s+1} \right)}{\tilde{p}_t(i) \frac{\prod_{l=1}^s \pi_{t+l}^{1-p}}{\prod_{l=1}^s \pi_{t+l}} - (1 - \delta_z) E_{t+s} \left( \frac{\bar{\beta} \gamma \xi_{t+s+1}}{\xi_{t+s}} \lambda_{t+s+1} \right)}$$

$$\begin{aligned}
& \frac{z_t(i)}{\omega_t(i)} (\theta_t - 1) \left[ \tilde{p}_t(i) - \frac{\theta_t}{\theta_t - 1} \left\{ \omega_t(i) \lambda_t + (1 - \omega_t(i))(1 - \delta_z) E_t \frac{\bar{\beta} \gamma \xi_{t+1}}{\xi_t} \lambda_{t+1} \right\} \right] \\
& + E_t \sum_{s=1}^{\infty} \xi_p^s \frac{\bar{\beta}^s \gamma^s \xi_{t+s}}{\xi_t} \frac{z_{t+s}(i)}{\omega_{t+s}(i)} (\theta_{t+s} - 1) \times \\
& \times \left[ \tilde{p}_t(i) \frac{\prod_{l=1}^s \pi_{t+l-1}^{\iota_p} \pi_*^{1-\iota_p}}{\prod_{l=1}^s \pi_{t+l}} - \frac{\theta_{t+s}}{\theta_{t+s} - 1} \left\{ \omega_{t+s}(i) \lambda_{t+s} + (1 - \omega_{t+s}(i))(1 - \delta_z) E_{t+s} \frac{\bar{\beta} \gamma \xi_{t+s+1}}{\xi_{t+s}} \lambda_{t+s+1} \right\} \right] \\
& = 0
\end{aligned}$$

where  $\omega_{t+s}(i) = \frac{v_{t+s}^*(i)}{\int \min(v, v_{t+s}^*(i)) dF(v)}$ .

$$\begin{aligned}
1 &= \int_0^1 p_t^{1-\theta_t}(i) \Phi(v_t^*(i)) di \\
&= (1 - \xi_p) \tilde{p}_t^{1-\theta_t}(i) \Phi(v_t^*(i)) + (1 - \xi_p) \sum_{s=1}^{\infty} \xi_p^s \tilde{p}_{t-s}^{1-\theta_t}(i) \left( \frac{\prod_{l=1}^s \pi_{t-l}^{\iota_p} \pi_*^{1-\iota_p}}{\prod_{l=1}^s \pi_{t-l+1}} \right)^{1-\theta_t} \Phi(v_t^*(i))
\end{aligned}$$

## B. Steady state

$$inv_* = \left( 1 - \frac{1 - \delta_z}{\gamma} \right) (z_* - s_*)$$

$$c_* + i_* + y_* g_* = s_*$$

$$s_* + inv_* = y_*$$

$$v_*^* = p_*^\theta \frac{z_*}{s_*}$$

$$\lambda_* = m c_*$$

$$1 - F(v_*^*) = \frac{(1 - (1 - \delta_z) \bar{\beta} \gamma) \lambda_*}{\tilde{p}_* - (1 - \delta_z) \bar{\beta} \gamma \lambda_*}$$

$$\tilde{p}_*^{1-\theta} \Phi \left( p_*^\theta \frac{z_*}{s_*} \right) = 1$$

$$\begin{aligned}
& \tilde{p}_* - \frac{\theta}{\theta - 1} \left( \frac{v_*^*}{\Psi \left( p_*^\theta \frac{z_*}{s_*} \right)} \lambda_* + \left( 1 - \frac{v_*^*}{\Psi \left( p_*^\theta \frac{z_*}{s_*} \right)} \right) (1 - \delta_z) \bar{\beta} \gamma \lambda_* \right) \\
&= \tilde{p}_* - \frac{\theta}{\theta - 1} (1 - \delta_z) \bar{\beta} \gamma \lambda_* - \frac{\theta}{\theta - 1} \frac{v_*^*}{\Psi \left( p_*^\theta \frac{z_*}{s_*} \right)} \lambda_* (1 - (1 - \delta_z) \bar{\beta} \gamma) = 0
\end{aligned}$$

or

$$\lambda_* = \frac{\theta - 1}{\theta} \frac{\tilde{p}_*}{\left( \frac{v_*^*}{\Psi \left( p_*^\theta \frac{z_*}{s_*} \right)} + \left( 1 - \frac{v_*^*}{\Psi \left( p_*^\theta \frac{z_*}{s_*} \right)} \right) (1 - \delta_z) \bar{\beta} \gamma \right)}$$

Log-linearized equations

$$inv_* \widehat{inv}_t = z_* \widehat{z}_t - s_* \widehat{s}_t - \frac{1 - \delta_z}{\gamma} (z_* \widehat{z}_{t-1} - s_* \widehat{s}_{t-1})$$

$$\frac{c_*}{y_*} \widehat{c}_t + \frac{i_*}{y_*} \widehat{i}_t + \widehat{g}_t + \frac{r_*^k k_*}{y_*} \widehat{u}_t = \frac{s_*}{y_*} \widehat{s}_t$$

$$\frac{s_*}{y_*} \widehat{s}_t + \frac{inv_*}{y_*} \widehat{inv}_t = \widehat{y}_t$$

First derive the equation for I-S ratios:

$$\begin{aligned}
\widehat{v}_{t+s}^*(i) &= \theta \left( \widehat{\tilde{p}}_t(i) + \sum_{l=1}^s [l_p \widehat{\pi}_{t+l-1} - \widehat{\pi}_{t+l}] \right) + \widehat{z}_{t+s}(i) - \widehat{s}_{t+s} + \widehat{\theta}_{t+s} \ln \tilde{p}_*^\theta \\
&= \theta \left( \widehat{\tilde{p}}_t(i) + \sum_{l=1}^s [l_p \widehat{\pi}_{t+l-1} - \widehat{\pi}_{t+l}] \right) + \widehat{z}_{t+s}(i) - \widehat{s}_{t+s} - \widehat{\lambda}_{p,t+s} \ln \tilde{p}_*^{\theta-1}
\end{aligned}$$

$$\widehat{\lambda}_{t+s}(i) = \widehat{m} \widehat{c}_{t+s} + \eta [\widehat{y}_{t+s}(i) - \widehat{y}_{t+s-1}(i)]$$

$$\begin{aligned}
1 - F(v_{t+s}^*(i)) &= \frac{mc_{t+s} - (1 - \delta_z)E_{t+s} \left( \frac{\bar{\beta}\gamma\xi_{t+s+1}}{\xi_{t+s}} mc_{t+s+1} \right)}{\tilde{p}_t(i) \frac{\prod_{l=1}^s \pi_{t+l}^{l_p} \pi_{t+l}^{1-l_p}}{\prod_{l=1}^s \pi_{t+l}} - (1 - \delta_z)E_{t+s} \left( \frac{\bar{\beta}\gamma\xi_{t+s+1}}{\xi_{t+s}} mc_{t+s+1} \right)} \\
&= \frac{\frac{f(v_*^*)v_*^*}{1 - F(v_*^*)} \left( \hat{s}_{t+s} - \hat{z}_{t+s}(i) - \theta \left( \hat{p}_t(i) + \sum_{l=1}^s [l_p \hat{\pi}_{t+l-1} - \hat{\pi}_{t+l}] \right) + \hat{\lambda}_{p,t+s} \ln \tilde{p}_*^{\theta-1} \right)}{\hat{\lambda}_{t+s} - (1 - \delta_z)\bar{\beta}\gamma E_{t+s} \left( \hat{\xi}_{t+s+1} - \hat{\xi}_{t+s} + \hat{\lambda}_{t+s+1} \right)} \\
&= \frac{\tilde{p}_* \left( \hat{p}_t(i) + \sum_{l=1}^s [l_p \hat{\pi}_{t+l-1} - \hat{\pi}_{t+l}] \right) - (1 - \delta_z)\bar{\beta}\gamma mc_* E_{t+s} \left( \hat{\xi}_{t+s+1} - \hat{\xi}_{t+s} + \hat{\lambda}_{t+s+1} \right)}{\tilde{p}_* - (1 - \delta_z)\bar{\beta}\gamma mc_*}
\end{aligned}$$

Aggregate and use aggregate price equation

$$\begin{aligned}
&\left\{ \hat{p}_t(i) + \sum_{s=1}^{\infty} \xi_p^s \left( \hat{p}_{t-s}(i) + \sum_{l=1}^s [l_p \hat{\pi}_{t-l} - \hat{\pi}_{t-l+1}] \right) \right\} (1 - \xi_p) \\
&= \hat{\lambda}_{p,t} \ln \tilde{p}_* - \frac{\tilde{p}_* \Phi'(v_*^*) v_*^*}{(1 - \theta) \Phi(v_*^*) + \theta \tilde{p}_* \Phi'(v_*^*) v_*^*} \left( \hat{z}_t - \hat{s}_t - \hat{\lambda}_{p,t} \ln \tilde{p}_*^{\theta-1} \right)
\end{aligned}$$

$$\begin{aligned}
&\frac{f(v_*^*)v_*^*}{1 - F(v_*^*)} \left( -\theta \hat{\lambda}_{p,t} \ln \tilde{p}_* + \left( \frac{\theta \tilde{p}_* \Phi'(v_*^*) v_*^*}{(1 - \theta) \Phi(v_*^*) + \theta \tilde{p}_* \Phi'(v_*^*) v_*^*} - 1 \right) \left( \hat{z}_t - \hat{s}_t - \hat{\lambda}_{p,t} \ln \tilde{p}_*^{\theta-1} \right) \right) \\
&= \frac{\hat{m}c_t - (1 - \delta_z)\bar{\beta}\gamma E_t \left( \hat{\xi}_{t+1} - \hat{\xi}_t + \hat{\lambda}_{t+1} \right)}{1 - (1 - \delta_z)\bar{\beta}\gamma} \\
&\quad \frac{\tilde{p}_* \left( \hat{\lambda}_{p,t} \ln \tilde{p}_* - \frac{\tilde{p}_* \Phi'(v_*^*) v_*^*}{(1 - \theta) \Phi(v_*^*) + \theta \tilde{p}_* \Phi'(v_*^*) v_*^*} \left( \hat{z}_t - \hat{s}_t - \hat{\lambda}_{p,t} \ln \tilde{p}_*^{\theta-1} \right) \right)}{\tilde{p}_* - (1 - \delta_z)\bar{\beta}\gamma mc_*} \\
&\quad + \frac{(1 - \delta_z)\bar{\beta}\gamma mc_* E_t \left( \hat{\xi}_{t+1} - \hat{\xi}_t + \hat{\lambda}_{t+1} \right)}{\tilde{p}_* - (1 - \delta_z)\bar{\beta}\gamma mc_*}
\end{aligned}$$

Let  $B_* = \frac{\tilde{p}_* \Phi'(v_*^*) v_*^*}{(1 - \theta) \Phi(v_*^*) + \theta \tilde{p}_* \Phi'(v_*^*) v_*^*}$ ,  $C_* = \frac{f(v_*^*)v_*^*}{1 - F(v_*^*)} \theta - \frac{\tilde{p}_*}{\tilde{p}_* - (1 - \delta_z)\bar{\beta}\gamma mc_*}$ . Simplify by gathering  $\hat{z}_t - \hat{s}_t -$

$\widehat{\lambda}_{p,t} \ln \widehat{p}_*^{\theta-1}$  term:

$$\begin{aligned} & \left( C_* B_* - \frac{f(v_*^*) v_*^*}{1 - F(v_*^*)} \right) \left( \widehat{z}_t - \widehat{s}_t - \widehat{\lambda}_{p,t} \ln \widehat{p}_*^{\theta-1} \right) = C_* \widehat{\lambda}_{p,t} \ln \widehat{p}_* \\ & + \frac{\widehat{m}c_t - (1 - \delta_z) \overline{\beta} \gamma E_t \left( \widehat{\xi}_{t+1} - \widehat{\xi}_t + \widehat{\lambda}_{t+1} \right)}{1 - (1 - \delta_z) \overline{\beta} \gamma} + \frac{(1 - \delta_z) \overline{\beta} \gamma m c_* E_t \left( \widehat{\xi}_{t+1} - \widehat{\xi}_t + \widehat{\lambda}_{t+1} \right)}{\widehat{p}_* - (1 - \delta_z) \overline{\beta} \gamma m c_*} \end{aligned}$$

Turning to the pricing equation:

$$\begin{aligned} & \frac{z_t(i)}{\omega_t(i)} \left[ \widetilde{p}_t(i) - \frac{\theta_t}{\theta_t - 1} \left\{ \omega_t(i) m c_t + (1 - \omega_t(i)) (1 - \delta_z) E_t \frac{\overline{\beta} \gamma \xi_{t+1}}{\xi_t} m c_{t+1} \right\} \right] \\ & + E_t \sum_{s=0}^{\infty} \xi_p^s \frac{\overline{\beta}^s \gamma^s \xi_{t+s}}{\xi_t} \frac{z_{t+s}(i)}{\omega_{t+s}(i)} \\ & = 0 \left[ \widetilde{p}_t(i) \frac{\prod_{l=1}^s \pi_{t+l-1}^{\iota_p} \pi_*^{1-\iota_p}}{\prod_{l=1}^s \pi_{t+l}} + \frac{\theta_t}{\theta_t - 1} \left\{ \omega_{t+s}(i) m c_{t+s} + (1 - \omega_{t+s}(i)) (1 - \delta_z) E_{t+s} \frac{\overline{\beta} \gamma \xi_{t+s+1}}{\xi_{t+s}} m c_{t+s+1} \right\} \right] \end{aligned}$$

where  $\omega_{t+s}(i) = \frac{v_{t+s}^*(i)}{\int \min(v, v_{t+s}^*(i)) dF(v)}$ . Next log-linearize the pricing equation:

$$\begin{aligned}
& \tilde{p}_* \widehat{p}_t(i) - \frac{\theta}{\theta-1} (1-\delta_z) \bar{\beta} \gamma m c_* E_t \left[ \widehat{\xi}_{t+1} - \widehat{\xi}_t + \widehat{\lambda}_{t+1} \right] \\
& - \frac{\theta}{\theta-1} \omega_* \left\{ m c_* \widehat{\lambda}_t - (1-\delta_z) \bar{\beta} \gamma m c_* E_t \left[ \widehat{\xi}_{t+1} - \widehat{\xi}_t + \widehat{\lambda}_{t+1} \right] \right\} \\
& - \frac{\theta}{\theta-1} \omega'_*(v_*) v_*^* m c_* (1 - (1-\delta_z) \bar{\beta} \gamma) \left( \theta \widehat{p}_t(i) + \widehat{z}_t(i) - \widehat{s}_t - \widehat{\lambda}_{p,t} \ln \tilde{p}_*^{\theta-1} \right) \\
& - \frac{\theta}{\theta-1} \left\{ \omega_* m c_* + (1-\omega_*) (1-\delta_z) \bar{\beta} \gamma m c_* \right\} \frac{1}{\theta} \widehat{\lambda}_{p,t} \\
& + E_t \sum_{s=1}^{\infty} \xi_p^s \bar{\beta}^s \gamma^s \left\{ \tilde{p}_* \widehat{p}_t(i) + \sum_{l=1}^s [l_p \widehat{\pi}_{t+l-1} - \widehat{\pi}_{t+l}] - \frac{\theta}{\theta-1} (1-\delta_z) \bar{\beta} \gamma m c_* \left[ \widehat{\xi}_{t+s+1} - \widehat{\xi}_{t+s} + \widehat{\lambda}_{t+s+1} \right] \right\} \\
& - \frac{\theta}{\theta-1} \omega_* \left\{ m c_* \widehat{\lambda}_{t+s} - (1-\delta_z) \bar{\beta} \gamma m c_* E_t \left[ \widehat{\xi}_{t+s+1} - \widehat{\xi}_{t+s} + \widehat{\lambda}_{t+s+1} \right] \right\} \\
& - \frac{\theta}{\theta-1} \left\{ \omega_* m c_* + (1-\omega_*) (1-\delta_z) \bar{\beta} \gamma m c_* \right\} \frac{1}{\theta} \widehat{\lambda}_{p,t+s} \\
& - \frac{\theta}{\theta-1} \omega'_*(v_*) v_*^* m c_* (1 - (1-\delta_z) \bar{\beta} \gamma) \times \\
& \quad \times \left( \theta \left( \widehat{p}_t(i) + \sum_{l=1}^s [l_p \widehat{\pi}_{t+l-1} - \widehat{\pi}_{t+l}] \right) + \widehat{z}_{t+s}(i) - \widehat{s}_{t+s} - \widehat{\lambda}_{p,t+s} \ln \tilde{p}_*^{\theta-1} \right) \Bigg\} \\
& = 0
\end{aligned}$$

where  $\omega_* = \frac{v_*^*}{\Psi(v_*^*)}$ ,  $\omega'_* = \frac{\Psi(v_*^*) - v_*^* \Psi'(v_*^*)}{[\Psi(v_*^*)]^2}$ ,

minus same at  $t+1$  multiplied by  $\xi_p \bar{\beta} \gamma$  gives

$$\begin{aligned}
& \widehat{p}_t(i) \left( \tilde{p}_* - \frac{\theta}{\theta-1} \omega'_*(v_*) v_*^* m c_* (1 - (1-\delta_z) \bar{\beta} \gamma) \theta \right) - \frac{\theta}{\theta-1} (1-\delta_z) \bar{\beta} \gamma m c_* E_t \left[ \widehat{\xi}_{t+1} - \widehat{\xi}_t + \widehat{\lambda}_{t+1} \right] \\
& - \frac{\theta}{\theta-1} \omega_* \left\{ m c_* \widehat{\lambda}_t - (1-\delta_z) \bar{\beta} \gamma m c_* E_t \left[ \widehat{\xi}_{t+1} - \widehat{\xi}_t + \widehat{\lambda}_{t+1} \right] \right\} \\
& - \frac{\theta}{\theta-1} \omega'_*(v_*) v_*^* m c_* (1 - (1-\delta_z) \bar{\beta} \gamma) \left( \widehat{z}_t - \widehat{s}_t - \widehat{\lambda}_{p,t} \ln \tilde{p}_*^{\theta-1} \right) \\
& - \frac{\theta}{\theta-1} \left\{ \omega_* m c_* + (1-\omega_*) (1-\delta_z) \bar{\beta} \gamma m c_* \right\} \frac{1}{\theta} \widehat{\lambda}_{p,t} \\
& - \frac{\xi_p \bar{\beta} \gamma}{1 - \xi_p \bar{\beta} \gamma} \left( \tilde{p}_* - \frac{\theta}{\theta-1} \omega'_*(v_*) v_*^* m c_* (1 - (1-\delta_z) \bar{\beta} \gamma) \theta \right) E_t \widehat{p}_{t+1}(i) \\
& + E_t \frac{\xi_p \bar{\beta} \gamma}{1 - \xi_p \bar{\beta} \gamma} \left( \left( \tilde{p}_* - \frac{\theta}{\theta-1} \omega'_*(v_*) v_*^* m c_* (1 - (1-\delta_z) \bar{\beta} \gamma) \theta \right) \widehat{p}_t(i) + [l_p \widehat{\pi}_t - \widehat{\pi}_{t+1}] \right) \\
& = 0
\end{aligned}$$

The aggregate price is determined by

$$\begin{aligned}
1 &= \int_0^1 p_t^{1-\theta}(i) \Phi(v_t^*(i)) di \\
&= (1 - \xi_p) \tilde{p}_t^{1-\theta}(i) \Phi(v_t^*(i)) + (1 - \xi_p) \sum_{s=1}^{\infty} \xi_p^s \tilde{p}_{t-s}^{1-\theta}(i) \left( \frac{\prod_{l=1}^s \pi_{t-l}^{\iota_p} \pi_*^{1-\iota_p}}{\prod_{l=1}^s \pi_{t-l+1}} \right)^{1-\theta} \Phi(v_t^*(i))
\end{aligned}$$

$$\begin{aligned}
0 &= (1 - \theta) \left[ \hat{p}_t(i) - \hat{\lambda}_{p,t} \ln \tilde{p}_* \right] \Phi(v_*^*) + \tilde{p}_*^{1-\theta} \Phi'(v_*^*) v_*^* \left( \theta \hat{p}_t(i) + \hat{z}_t(i) - \hat{s}_t - \hat{\lambda}_{p,t} \ln \tilde{p}_*^{\theta-1} \right) \\
&+ (1 - \theta) \sum_{s=1}^{\infty} \xi_p^s \left( \hat{p}_{t-s}(i) + \sum_{l=1}^s [\iota_p \hat{\pi}_{t-l} - \hat{\pi}_{t-l+1}] - \hat{\lambda}_{p,t} \ln \tilde{p}_* \right) \Phi(v_*^*) \\
&+ \sum_{s=1}^{\infty} \xi_p^s \left[ \tilde{p}_*^{1-\theta} \Phi'(v_*^*) v_*^* \left( \theta \hat{p}_{t-s}(i) + \theta \sum_{l=1}^s [\iota_p \hat{\pi}_{t-l} - \hat{\pi}_{t-l+1}] + \hat{z}_t(i) - \hat{s}_t - \hat{\lambda}_{p,t} \ln \tilde{p}_*^{\theta-1} \right) \right]
\end{aligned}$$

$$\begin{aligned}
0 &= \hat{p}_t(i) - \hat{\lambda}_{p,t} \ln \tilde{p}_* + \frac{1}{1 - \xi_p} \frac{\tilde{p}_* \Phi'(v_*^*) v_*^*}{(1 - \theta) \Phi(v_*^*) + \theta \tilde{p}_* \Phi'(v_*^*) v_*^*} \left( \hat{z}_t - \hat{s}_t - \hat{\lambda}_{p,t} \ln \tilde{p}_*^{\theta-1} \right) \\
&+ \sum_{s=1}^{\infty} \xi_p^s \left( \hat{p}_{t-s}(i) + \sum_{l=1}^s [\iota_p \hat{\pi}_{t-l} - \hat{\pi}_{t-l+1}] - \hat{\lambda}_{p,t} \ln \tilde{p}_* \right)
\end{aligned}$$

minus same at  $t - 1$  multiplied by  $\xi_p$  :

$$\begin{aligned}
\hat{p}_t(i) - \frac{1}{1 - \xi_p} \hat{\lambda}_{p,t} \ln \tilde{p}_* &= \frac{\xi_p}{1 - \xi_p} [\hat{\pi}_t - \iota_p \hat{\pi}_{t-1}] \\
&- \frac{1}{1 - \xi_p} \frac{\tilde{p}_* \Phi'(v_*^*) v_*^*}{(1 - \theta) \Phi(v_*^*) + \theta \tilde{p}_* \Phi'(v_*^*) v_*^*} \left[ \left( \hat{z}_t - \hat{s}_t - \hat{\lambda}_{p,t} \ln \tilde{p}_*^{\theta-1} \right) - \xi_p \left( \hat{z}_{t-1} - \hat{s}_{t-1} - \hat{\lambda}_{p,t-1} \ln \tilde{p}_*^{\theta-1} \right) \right]
\end{aligned}$$

Let  $A_* = \tilde{p}_* - \frac{\theta}{\theta-1} \omega'_*(v_*) v_*^* m c_*(1 - (1 - \delta_z) \bar{\beta} \gamma) \theta$ . Plugging this equation back into the pricing equation yields the NKPC:

$$\begin{aligned}
& \frac{1}{1 - \xi_p \bar{\beta} \gamma} \frac{\xi_p}{1 - \xi_p} \left[ \hat{\pi}_t - \iota_p \hat{\pi}_{t-1} + \frac{1}{\xi_p} \hat{\lambda}_{p,t} \ln \tilde{p}_*^{\theta-1} \right] A_* \\
& - \frac{1}{1 - \xi_p \bar{\beta} \gamma} \frac{1}{1 - \xi_p} B_* \left[ \left( \hat{z}_t - \hat{s}_t - \hat{\lambda}_{p,t} \ln \tilde{p}_*^{\theta-1} \right) - \xi_p \left( \hat{z}_{t-1} - \hat{s}_{t-1} - \hat{\lambda}_{p,t-1} \ln \tilde{p}_*^{\theta-1} \right) \right] A_* \\
& - \frac{\theta}{\theta - 1} (1 - \delta_z) \bar{\beta} \gamma m c_* E_t \left[ \hat{\xi}_{t+1} - \hat{\xi}_t + \hat{\lambda}_{t+1} \right] \\
& - \frac{\theta}{\theta - 1} \omega_* \left\{ m c_* \hat{\lambda}_t - (1 - \delta_z) \bar{\beta} \gamma m c_* E_t \left[ \hat{\xi}_{t+1} - \hat{\xi}_t + \hat{\lambda}_{t+1} \right] \right\} \\
& - \frac{\theta}{\theta - 1} \left\{ \omega_* m c_* + (1 - \omega_*) (1 - \delta_z) \bar{\beta} \gamma m c_* \right\} \frac{1}{\theta} \hat{\lambda}_{p,t} \\
& - \frac{\theta}{\theta - 1} \omega'_* (v_*^*) v_*^* m c_* (1 - (1 - \delta_z) \bar{\beta} \gamma) \left( \hat{z}_t - \hat{s}_t - \hat{\lambda}_{p,t} \ln \tilde{p}_*^{\theta-1} \right) \\
& - \frac{\xi_p \bar{\beta} \gamma}{1 - \xi_p \bar{\beta} \gamma} A_* E_t \left\{ \begin{array}{l} \frac{\xi_p}{1 - \xi_p} \left[ \hat{\pi}_{t+1} - \iota_p \hat{\pi}_t + \frac{1}{\xi_p} \hat{\lambda}_{p,t+1} \ln \tilde{p}_*^{\theta-1} \right] \\ - \frac{1}{1 - \xi_p} B_* \left[ \left( \hat{z}_{t+1} - \hat{s}_{t+1} - \hat{\lambda}_{p,t+1} \ln \tilde{p}_*^{\theta-1} \right) - \xi_p \left( \hat{z}_t - \hat{s}_t - \hat{\lambda}_{p,t} \ln \tilde{p}_*^{\theta-1} \right) \right] \end{array} \right\} \\
& + E_t \frac{\xi_p \bar{\beta} \gamma}{1 - \xi_p \bar{\beta} \gamma} \left[ \iota_p \hat{\pi}_t - \hat{\pi}_{t+1} \right] \\
& = 0
\end{aligned}$$

or after simplification

$$\begin{aligned}
& \hat{\pi}_t - \iota_p \hat{\pi}_{t-1} \\
& = \frac{1 - \xi_p (1 - A_*)}{\bar{\beta} \gamma A_*} E_t \left[ \hat{\pi}_{t+1} - \iota_p \hat{\pi}_t \right] \\
& + \frac{(1 - \xi_p \bar{\beta} \gamma) (1 - \xi_p)}{\xi_p A_*} \left[ \frac{\theta}{\theta - 1} \omega_* m c_* \hat{\lambda}_t - (\omega_* - 1) \frac{\theta}{\theta - 1} (1 - \delta_z) \bar{\beta} \gamma m c_* E_t \left[ \hat{\xi}_{t+1} - \hat{\xi}_t + \hat{\lambda}_{t+1} \right] \right] \\
& - B_* \left( \hat{z}_{t-1} - \hat{s}_{t-1} - \hat{\lambda}_{p,t-1} \ln \tilde{p}_*^{\theta-1} \right) \\
& + \left\{ \frac{(1 - \xi_p \bar{\beta} \gamma) (1 - \xi_p)}{\xi_p \theta} \frac{\tilde{p}_* - A_*}{A_*} + \frac{B_*}{\xi_p} + \xi_p \bar{\beta} \gamma B_* \right\} \left( \hat{z}_t - \hat{s}_t - \hat{\lambda}_{p,t} \ln \tilde{p}_*^{\theta-1} \right) \\
& - \bar{\beta} \gamma E_t \left( B_* (\hat{z}_{t+1} - \hat{s}_{t+1}) - (B_* + 1) \hat{\lambda}_{p,t+1} \ln \tilde{p}_*^{\theta-1} \right) \\
& + \left[ -\frac{1}{\xi_p} \ln \tilde{p}_*^{\theta-1} + \frac{\theta}{\theta - 1} \left\{ \omega_* m c_* + (1 - \omega_*) (1 - \delta_z) \bar{\beta} \gamma m c_* \right\} \frac{1}{\theta} \frac{(1 - \xi_p \bar{\beta} \gamma) (1 - \xi_p)}{\xi_p A_*} \right] \hat{\lambda}_{p,t}
\end{aligned}$$

### C. Summary

Modify resource constraint:

$$inv_* = \left(1 - \frac{1 - \delta_z}{\gamma}\right) (z_* - s_*)$$

$$c_* + i_* + y_* g_* = s_*$$

$$s_* + inv_* = y_*$$

Add steady state equations for  $\frac{z_*}{s_*}$ ,  $mc_*$  and  $\tilde{p}_*$ :

$$1 - F\left(\tilde{p}_*^\theta \frac{z_*}{s_*}\right) = \frac{(1 - (1 - \delta_z)\bar{\beta}\gamma) mc_*}{\tilde{p}_* - (1 - \delta_z)\bar{\beta}\gamma mc_*} \quad (4)$$

$$\tilde{p}_*^{1-\theta} \Phi\left(\tilde{p}_*^\theta \frac{z_*}{s_*}\right) = 1 \quad (5)$$

$$\begin{aligned} & \tilde{p}_* - \frac{\theta}{\theta - 1} \left( \frac{v_*^*}{\Psi\left(p_*^\theta \frac{z_*}{s_*}\right)} \lambda_* + \left(1 - \frac{v_*^*}{\Psi\left(p_*^\theta \frac{z_*}{s_*}\right)}\right) (1 - \delta_z)\bar{\beta}\gamma \lambda_* \right) \\ &= \tilde{p}_* - \frac{\theta}{\theta - 1} (1 - \delta_z)\bar{\beta}\gamma \lambda_* - \frac{\theta}{\theta - 1} \frac{v_*^*}{\Psi\left(p_*^\theta \frac{z_*}{s_*}\right)} \lambda_* (1 - (1 - \delta_z)\bar{\beta}\gamma) = 0 \\ \lambda_* &= \frac{\theta - 1}{\theta} \frac{\tilde{p}_*}{\left( \frac{v_*^*}{\Psi\left(p_*^\theta \frac{z_*}{s_*}\right)} + \left(1 - \frac{v_*^*}{\Psi\left(p_*^\theta \frac{z_*}{s_*}\right)}\right) (1 - \delta_z)\bar{\beta}\gamma \right)} \quad (6) \end{aligned}$$

Equations (4)-(6) are solved jointly for  $\frac{z_*}{s_*}$ ,  $\lambda_*$  and  $\tilde{p}_*$ .

Denote

$$\begin{aligned}
v_*^* &= \tilde{p}_*^\theta \frac{z_*^*}{s_*^*} \\
\Phi'(v^*) &= \frac{\theta - 1}{\theta} (v^*)^{\frac{-1}{\theta}} \exp\left(\frac{\sigma^2}{2\theta^2}\right) (1 - F(v^* - \sigma^2/\theta)) \\
\Psi(v^*) &= \int \min(v, v_t^*(i)) dF(v) \\
&= \exp\left(\frac{\sigma^2}{2}\right) F(v_t^*(i) - \sigma^2) + v_t^*(i) (1 - F(v_t^*(i))) \\
\Psi'(v^*) &= 1 - F(v_t^*(i)) \\
\omega(v^*) &= \frac{v^*}{\Psi(v^*)} \\
\omega'(v^*) &= \frac{\Psi(v^*) - v^* \Psi'(v^*)}{[\Psi(v^*)]^2}
\end{aligned}$$

$$\begin{aligned}
A_* &= \tilde{p}_* - \frac{\theta}{\theta - 1} \omega'(v_*^*) v_*^* m c_* (1 - (1 - \delta_z) \bar{\beta} \gamma) \theta \\
B_* &= \frac{\tilde{p}_* \Phi'(v_*^*) v_*^*}{(1 - \theta) \Phi(v_*^*) + \theta \tilde{p}_* \Phi'(v_*^*) v_*^*} \\
C_* &= \frac{f(v_*^*) v_*^*}{1 - F(v_*^*)} \theta - \frac{\tilde{p}_*}{\tilde{p}_* - (1 - \delta_z) \bar{\beta} \gamma m c_*}
\end{aligned}$$

Add log-linearized resource constraint for  $\widehat{inv}_t$  and  $\widehat{z}_t$ :

$$\widehat{inv}_* \widehat{inv}_t = z_* \widehat{z}_t - s_* \widehat{s}_t - \frac{1 - \delta_z}{\gamma} (z_* \widehat{z}_{t-1} - s_* \widehat{s}_{t-1})$$

$$\frac{c_*}{y_*} \widehat{c}_t + \frac{i_*}{y_*} \widehat{i}_t + \widehat{g}_t + \frac{r_*^k k_*}{y_*} \widehat{u}_t = \frac{s_*}{y_*} \widehat{s}_t$$

$$\frac{s_*}{y_*} \widehat{s}_t + \frac{\widehat{inv}_*}{y_*} \widehat{inv}_t = \widehat{y}_t$$

Log-linearized equations for I-S ratio

$$\widehat{\lambda}_{t+s}(i) = \widehat{m}c_{t+s} + \eta \widehat{y}_{t+s}(i)$$

$$\begin{aligned} & \left( C_* B_* - \frac{f(v_*) v_*}{1 - F(v_*)} \right) \left( \widehat{z}_t - \widehat{s}_t - \widehat{\lambda}_{p,t} \ln \widetilde{p}_*^{\theta-1} \right) = C_* \widehat{\lambda}_{p,t} \ln \widetilde{p}_* \\ & + \frac{\widehat{\lambda}_t - (1 - \delta_z) \bar{\beta} \gamma E_t \left( \widehat{\xi}_{t+1} - \widehat{\xi}_t + \widehat{\lambda}_{t+1} \right)}{1 - (1 - \delta_z) \bar{\beta} \gamma} + \frac{(1 - \delta_z) \bar{\beta} \gamma m c_* E_t \left( \widehat{\xi}_{t+1} - \widehat{\xi}_t + \widehat{\lambda}_{t+1} \right)}{\widetilde{p}_* - (1 - \delta_z) \bar{\beta} \gamma m c_*} \end{aligned}$$

and NKPC:

$$\begin{aligned} & \widehat{\pi}_t - \iota_p \widehat{\pi}_{t-1} \\ = & \bar{\beta} \gamma \frac{1 - \xi_p (1 - A_*)}{A_*} E_t \left[ \widehat{\pi}_{t+1} - \iota_p \widehat{\pi}_t \right] \\ & + \frac{(1 - \xi_p \bar{\beta} \gamma) (1 - \xi_p)}{\xi_p A_*} \left[ \frac{\theta}{\theta - 1} \omega_* m c_* \widehat{\lambda}_t - (\omega_* - 1) \frac{\theta}{\theta - 1} (1 - \delta_z) \bar{\beta} \gamma m c_* E_t \left[ \widehat{\xi}_{t+1} - \widehat{\xi}_t + \widehat{\lambda}_{t+1} \right] \right] \\ & - B_* \left( \widehat{z}_{t-1} - \widehat{s}_{t-1} - \widehat{\lambda}_{p,t-1} \ln \widetilde{p}_*^{\theta-1} \right) \\ & + \left\{ \frac{(1 - \xi_p \bar{\beta} \gamma) (1 - \xi_p)}{\xi_p \theta} \frac{\widetilde{p}_* - A_*}{A_*} + \frac{B_*}{\xi_p} + \xi_p \bar{\beta} \gamma B_* \right\} \left( \widehat{z}_t - \widehat{s}_t - \widehat{\lambda}_{p,t} \ln \widetilde{p}_*^{\theta-1} \right) \\ & - \bar{\beta} \gamma E_t \left( B_* (\widehat{z}_{t+1} - \widehat{s}_{t+1}) - (B_* + 1) \widehat{\lambda}_{p,t+1} \ln \widetilde{p}_*^{\theta-1} \right) \\ & + \left[ -\frac{1}{\xi_p} \ln \widetilde{p}_*^{\theta-1} + \frac{\theta}{\theta - 1} \{ \omega_* m c_* + (1 - \omega_*) (1 - \delta_z) \bar{\beta} \gamma m c_* \} \frac{1}{\theta} \frac{(1 - \xi_p \bar{\beta} \gamma) (1 - \xi_p)}{\xi_p A_*} \right] \widehat{\lambda}_{p,t} \end{aligned}$$